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Generalized autoregressive conditional heteroscedasticity modelling of hydrologic time series

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Abstract:

The existence of time-dependent variance or conditional variance, commonly called heteroscedasticity, in hydrologic time series has not been thoroughly investigated. This paper deals with modelling the heteroscedasticity in the residuals of the seasonal autoregressive integrated moving average (SARIMA) model using a generalized autoregressive conditional heteroscedasticity (GARCH) model. The model is applied to two monthly rainfall time series from humid and arid regions. The effect of Box–Cox transformation and seasonal differencing on the remaining seasonal heteroscedasticity in the residuals of the SARIMA model is also investigated. It is shown that the seasonal heteroscedasticity in the residuals of the SARIMA model can be removed using Box–Cox transformation along with seasonal differencing for the humid region rainfall. On the other hand, transformation and seasonal differencing could not remove heteroscedasticity from the residuals of the SARIMA model fitted to rainfall data in the arid region. Therefore, the GARCH modelling approach is necessary to capture the heteroscedasticity remaining in the residuals of a SARIMA model. However, the evaluation criteria do not necessarily show that the GARCH model improves the performance of the SARIMA model. Copyright © 2012 John Wiley & Sons, Ltd.

KEY WORDS nonlinear time series; heteroscedasticity; GARCH; Engle's test; SARIMA model; seasonality; Box–Cox transformation

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INTRODUCTION

During the last decades, hydrological time series analysis and modelling have received considerable attention for hydrologic prediction, simulation and forecasting. One of the main concerns of hydrological time series modelling is whether the hydrologic variable is governed by a linear or a nonlinear process through time.

The application of the linear time series models has been widely discussed in the literature for different hydrologic and climatic variables such as rainfall (Astel *et al*., 2004; Machiwal and Jha, 2008), streamflows (Ouarda *et al*., 1997; Adeloye and Montaseri, 2002), floods (Toth *et al*., 1999), droughts (Modarres, 2007) and water quality variables (Worrall and Burt, 1999).

Although there is growing interest in nonlinear analysis of hydrologic time series, relatively few efforts have been done in this field. The generalized autoregressive conditional heteroscedasticity (GARCH) approach, which is commonly used in modelling the time variation of the second order moment or the variance of financial time series, can be an appropriate method for nonlinear modelling of hydrologic time series. Although the time variation of the variance of hydrologic variables has been mentioned in the literature, few studies have applied the GARCH approach to model this phenomenon in hydrologic variables.

In a pioneering study, Wang *et al*. (2005) applied the GARCH approach to model the heteroscedasticity of daily and monthly streamflow time series of the upper Yellow River at Tangnaihai in China. They concluded that the conventional linear time series models, the autoregressive model and the deseasonalized autoregressive moving average (ARMA) model, are not sufficient to describe the time-dependent variance of streamflow and that a GARCH model needs to be fitted to the residuals of an ARMA model to capture the time variation behaviour of the streamflow variance. Chen *et al*. (2008) applied linear ARMA and nonlinear ARCH models to model 10-day streamflows of the Wu-Shi River in Taiwan and verified that nonlinear time series models are superior to the traditional linear approaches such as an ARMA model. They reported an increase of the coefficient of efficiency (CE) from 0.28 for an ARMA model to 0.76 for an ARCH model whereas the mean absolute error (MAE) was reduced from 60.45 for an ARMA model to 41.35 for an ARCH model. However, no seasonal or integrated time series model was applied to compare with a GARCH model in their study.

In another application of the GARCH model, Romilly (2005) fitted a seasonal autoregressive integrated moving average (SARIMA) model to the global mean monthly temperature and mentioned the existence of heteroscedasticity in the residuals and the need for applying a GARCH model to remove it from the residuals. However, the model comparison in this study indicated that the SARIMA model performs slightly better than the GARCH model for the global mean temperature time series

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modelling according to $R^2 = 0.772$, mean absolute error (MAE = 0.084) and the root mean squared error (RMSE = 0.111) for the SARIMA model against $R^2 = 0.771$, MAE = 0.084 and RMSE = 0.112 for the GARCH model.

In addition to the fact of rare GARCH model applications in hydrology, rainfall is a missing variable in heteroscedastic time series modelling. Rainfall plays a vital role in the successful development and implementation of water resource tools to assess engineering and environmental problems such as flood control, reservoir operation, hydropower generation and water quality control. Henceforth, the efficient rainfall estimation, modelling and forecasting are a critical mission in hydrology, water resources and environmental sciences.

The main objective of this study is, therefore, to illustrate the advantages/disadvantages of a GARCH approach for modelling the monthly rainfall time series against the SARIMA model commonly used for monthly hydrologic time series modelling. In addition, the effect of Box–Cox transformation on stabilizing the variance of rainfall data and its effect on the advantages/disadvantages of a GARCH model are also investigated.

METHODS

SARIMA model

The SARIMA time series model has a general multiplicative form, SARIMA(p,d,q) × (P,D,Q) (Hipel and McLeod, 1996). The first set of brackets contains the order of the nonseasonal parameters whereas the orders of the seasonal parameters are listed inside the second set of brackets.

The mathematical formulation of a SARIMA(p,d,q) × (P,D,Q) model can be written as follows:

$$\phi_p(B)\Phi_P\left(B^S\right)\nabla^d\nabla_S{}^D Y_t = \theta_q(B)\Theta_Q\left(B^S\right)a_t \qquad (1)$$

where Y_t is the observed time series, $\phi_p(B)$ is a polynomial of order p, $\theta_q(B)$ is a polynomial of order q, Φ_P and Θ_Q are the seasonal polynomials of order P and Q, ∇^d and ∇_S^D are the nonseasonal and seasonal differencing operators, respectively, B is the backward operator, S represents the number of seasons per year and a_t is an independent identically distributed (i.i.d.) normal error with a zero mean and standard deviation σ_a, respectively.

The SARIMA model in Equation (1) is referred to as a multiplicative model, as the nonseasonal and seasonal autoregressive operators are multiplied together on the left-hand side whereas the two moving average operators are multiplied together on the right-hand side (Hipel and McLeod, 1996). Building the above SARIMA model from the observations includes three steps: model identification, parameter estimation and goodness-of-fit test or checking model adequacy. When the order of parameters of an initial model is identified according to the structure of the autocorrelation function (ACF) and its (significant) parameters are estimated using a method of estimation such as the method of moments or the method of maximum likelihood, the model adequacy should be checked. The details of model building can be found in Hipel and McLeod (1996).

For checking the adequacy of a model, the ACF of the residuals of a SARIMA model (ε) is first inspected. It is well known that for random and independent series of length n, the lag k autocorrelation coefficient is normally distributed with a mean zero and a variance $1/n$, and the 95% confidence limits are given by $\pm 1.96/\sqrt{n}$. If all autocorrelation coefficients fall within the confidence limits, the adequacy of the time series model is accepted.

More formally, the Ljung–Box lack-of-fit test (commonly called the portmanteau lack-of-fit test) is also used to test the adequacy of the SARIMA model. The Portmanteau lack-of-fit-test (Ljung and Box, 1978) computes a statistic Q, which is χ^2distributed on $(L-p-q)$ degrees of freedom and is given by

$$Q = N(N+2)\sum_{k=1}^{L}(N-k)^{-1}r_k^2(\varepsilon) \qquad (2)$$

where N is the sample size; L is the number of autocorrelations of the residuals included in the statistic, which can be 15 to 25 for nonseasonal models and 2 S to 4 S for seasonal models (Hipel and McLeod, 1996); and r_k is the sample autocorrelation of the residual time series, ε, at lag k. If the probability of Q is higher than $\alpha = 0.05$, there is strong evidence that the residuals are time independent and the model is adequate. If this probability is less than $\alpha = 0.05$, it is reasonable to conclude that the residuals are time dependent and the model is inadequate, and we need to repeat the process of model building to achieve an adequate model (Hipel and McLeod, 1996).

GARCH modelling approach

By applying a SARIMA model, the dependence of an observation at time t, Y_t, to the previous observations, $Y_{t-1}, Y_{t-2}, \ldots, Y_{t-p}$, or the conditional mean, is considered for time series modelling. Although the errors or the residuals of a SARIMA model may exhibit adequacy, conditional variance or time-dependent variance may exist in the residuals, which can be captured by a GARCH model. This type of model is also called a SARIMA–GARCH error model, as the conditional variance of the residuals of a SARIMA model will be modelled by a GARCH approach. To describe a GARCH modelling approach, the ARCH model is first described.

If the conditional mean is described by a SARIMA model, the Vth order of the ARCH(V) model for the conditional variance of the residuals of a SARIMA model, (σ_t), is defined as follows (Engle, 1982):

$$\begin{aligned}\sigma_t^2 &= \omega + \alpha_1\varepsilon_{t-1}^2 + \alpha_2\varepsilon_{t-2}^2 + \ldots + \alpha_V\varepsilon_{t-V}^2 \\ &= \omega + \sum_{i=1}^{V}\alpha_V\varepsilon_{t-V}^2\end{aligned} \qquad (3)$$

$$\varepsilon_t = \sigma_t e_t \quad e_t \sim \text{Normal}(0, 1) \tag{4}$$

where ε_t indicates the residuals of the SARIMA model that are uncorrelated but have variances that change over time; e_t denotes a real valued i.i.d. random variable with mean 0 and variance 1, independent of past realizations $(e_{t-1}, e_{t-2}, \ldots)$; $\alpha_1, \ldots, \alpha_V$ are the parameters of the ARCH model; and ω is a constant (Wei, 2006). In this model, the variance of the error is time varying and depends on the V past errors, ε^2_{t-1}, ε^2_{t-2},..., ε^2_{t-V} through their squares.

The GARCH model, introduced by Bollerslev (1986), improves the original specification by adding lagged conditional variance, which acts as a smoothing term.

The GARCH(V,M) model is defined as follows:

$$\sigma_t^2 = \omega + \sum_{i=1}^{V} \alpha_V \varepsilon^2_{t-V} + \sum_{j=1}^{M} \beta_M \sigma^2_{t-M} \tag{5}$$

$$\varepsilon_t = \sigma_t e_t \quad e_t \sim \text{Normal}(0, 1) \tag{6}$$

where $\alpha_1, \ldots, \alpha_V$ and $\beta_1, \ldots, \beta_M$ are the parameters of the GARCH(V,M) process. Therefore, the SARIMA–GARCH(V,M) model is one in which the conditional mean is described by a SARIMA model whereas the conditional variance is described by a GARCH(V,M) model.

Tests for the ARCH effect

Although the ACF of the residuals and the Ljung–Box lack-of-fit test are usually used for time series model adequacy analysis, the variance time-dependent characteristics of the residuals cannot be inspected using the ACF of the residuals. Bollerslev (1986) stated that the ACF of the standardized squared residuals (SSRs, hereafter) is useful for identifying and checking the heteroscedasticity of residuals. In order to check if the variance of the residuals is conditional on its history or, in other words, if the residuals exhibit an ARCH effect, we apply the commonly used Ljung–Box and Engle's Lagrange Multiplier tests for the SSRs.

The Ljung–Box lack-of-fit test for SSRs is calculated through Equation (2) but for the SSR time series (ε^2):

$$Q = N(N+2) \sum_{k=1}^{L} (N-k)^{-1} r_k^2\left(\varepsilon^2\right) \tag{7}$$

Under the null hypothesis of no ARCH effect in the residuals, the test statistic is asymptotically chi-squared distributed.

The Engle's Lagrange Multiplier test for the ARCH effect was proposed by Engle (1982). The test statistic is given by NR^2, where R is the sample multiple correlation coefficient computed from the regression of ε_t^2 on a constant and $\varepsilon^2_{t-1}, \ldots, \varepsilon^2_{t-V}$ and N is the sample size. The null hypothesis of no ARCH effect is accepted if the test statistic is asymptotically distributed as a chi-squared distribution with v degrees of freedom. The test can also be used to investigate the GARCH effect of SSRs (Bollerslev, 1986).

Data transformation

Although a few studies have applied GARCH models in hydrology, the existence of conditional variance in hydrologic variables has already been addressed for a long time, and some methods have been used to stabilize this variance. The Box–Cox class of power transformation technique (Box and Cox, 1964) is the most common method for variance stabilization. Box–Cox transformation can be expressed by the following equation:

$$Y_i^{B-C} = \begin{cases} \dfrac{Y_i^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \log(Y_i) & \text{if } \lambda = 0 \end{cases} \tag{8}$$

where Y_i^{B-C} is the Box–Cox transformed data, Y_i is the original time series and λ is the power parameter chosen to ensure that the transformed data are approximately Gaussian. Box–Cox transformation is assumed to stabilize the variance of a time series so that no further ARCH effect is observed in the residuals of a time series model. The optimum value of λ is chosen based on a Box–Cox normality plot, which indicates the variation of λ against the correlation coefficient of a normal probability plot. The value of λ corresponding to the maximum correlation is then the optimal choice for λ (Lye, 1993).

Comparison approach

The time series modelling approaches used in this study are compared through a multi-criteria comparison by applying a set of evaluation metrics (Modarres, 2009). The evaluation metrics can be classified into three groups: metrics that calculate the absolute error, metrics that calculate the relative error and the dimensionless metrics (Dawson *et al.*, 2007). In this study, the following evaluation metrics of the above categories are used:

1. Metrics for calculating absolute errors

1.1 Absolute maximum error (AME)

$$\text{AME} = \max\left(\left|Q_i - \hat{Q}_i\right|\right) \tag{9}$$

1.2 Peak difference (PDIFF)

$$\text{PDIFF} = \max(Q_i) - \max\left(\hat{Q}_i\right) \tag{10}$$

1.3 Mean absolute error (MAE)

$$\text{MAE} = \frac{1}{n} \sum \left|Q_i - \hat{Q}_i\right| \tag{11}$$

1.4 Root mean squared error (RMSE)

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}\left(Q_i - \hat{Q}_i\right)^2}{n}} \quad (12)$$

2. Metrics for calculating relative errors

2.1 Relative absolute error

$$\mathrm{RAE} = \frac{\sum_{i=1}^{n}\left|Q_i - \hat{Q}_i\right|}{\sum_{i=1}^{n}\left|Q_i - \bar{Q}_i\right|} \quad (13)$$

2.2 Mean relative error

$$\mathrm{MRE} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{Q_i - \hat{Q}_i}{Q_i}\right) \quad (14)$$

3. Dimensionless metrics

3.1 Coefficient of determination (R-squared)

$$R^2 = \left[\frac{\sum_{i=1}^{n}\left(Q_i - \bar{Q}\right)\left(\hat{Q}_i - \tilde{Q}\right)}{\sqrt{\sum_{i=1}^{n}\left(Q_i - \bar{Q}\right)^2 \sum_{i=1}^{n}\left(\hat{Q}_i - \tilde{Q}\right)^2}}\right]^2 \quad (15)$$

3.2 Coefficient of efficiency (CE)

$$\mathrm{CE} = 1 - \frac{\sum_{i=1}^{n}\left(Q_i - \hat{Q}_i\right)^2}{\sum_{i=1}^{n}\left(Q_i - \bar{Q}\right)^2} \quad (16)$$

3.3 Index of agreement (IoAd)

$$\mathrm{IoAd} = 1 - \frac{\sum_{i=1}^{n}\left(Q_i - \hat{Q}_i\right)^2}{\sum_{i=1}^{n}\left(\left|\hat{Q}_i - \bar{Q}\right| + \left|Q_i - \bar{Q}\right|\right)^2} \quad (17)$$

In these equations, $Q_i$is the observed time series, $\hat{Q}_i$is the predicted time series, and $\bar{Q}$ and $\tilde{Q}$are the mean of the observed and predicted time series, respectively.

Although the above criteria allow us to sort models based on their accuracy, it is also important to test whether the improvement of model accuracy between two competing models is statistically significant. To address this issue, we use a test statistic proposed by Diebold and Mariano (1995), which is already used in financial time series but is applied and reported for the first time in hydrological sciences. Although this test is usually used for out-of-sample forecasting accuracy analysis (i.e. Mohammadi and Su, 2010), here, we use it for within-sample prediction accuracy testing. In our case, this test can indicate if there is a significant difference between SARIMA and SARIMA–GARCH's performances for predicting (estimating) rainfall time series.

Formally, let $e_{1,t}$ and $e_{2,t}$, $t=1, \ldots, n$, denote model errors from SARIMA an SARIMA–GARCH models and $g(e_{1,t})$ and $g(e_{2,t})$ are their associated loss functions and $d_t = g(e_{1,t}) - g(e_{2,t})$. Diebold and Mariano (1995) defined the B statistics:

$$B = \frac{\bar{d}}{\sqrt{\frac{s}{n}}} \quad \sim N(0,1) \quad (18)$$

where $\bar{d}$ is the sample mean, s is the variance of loss differential and n is the number of observations. Under a null hypothesis of zero mean loss differential, the null hypothesis of equal modelling accuracy is rejected if the test statistic is negative and statistically significant.

APPLICATIONS AND CASE STUDIES

In this section, we illustrate the application of the above methods for two monthly rainfall time series, one from a humid region and the other from an arid region. These two rainfall data sets are selected to investigate and compare the ARCH effect of rainfall in the two regions and to evaluate the GARCH model efficiency and advantage for modelling the rainfall heteroscedasticity in the humid and arid regions.

Rainfall for a humid region

Data. The first case study includes the monthly time series of the Campsie station (54°07'N and 114°40'W), Alberta province (station elevation: 670.6 m), Canada. The mean annual rainfall in the Campsie station is 343 mm for the period of 1950–2008. The monthly rainfall time series of the Campsie station is illustrated in Figure 1a. The monthly distribution of the mean and standard deviation (STDEV) of rainfall (Figure 1b) indicates seasonality in both rainfall depth and variance in the Campsie station. In the following sections, the results of time series models fitted to both original and transformed data are given and discussed.

Results for modelling original data. Because the ACF of rainfall time series (Figure 2a) indicates a strong seasonal structure, the SARIMA$(1,0,1)_{12}$ model is fitted to the rainfall data. This model includes two seasonal autoregressive and moving average parameters of order 1 that are significant at the 95% level and do not include any nonseasonal parameters.

The adequacy of the SARIMA$(1,0,1)_{12}$ model is verified using the Ljung–Box statistics and the ACF of the residuals. The ACF of the residuals (Figure 2b) indicates no

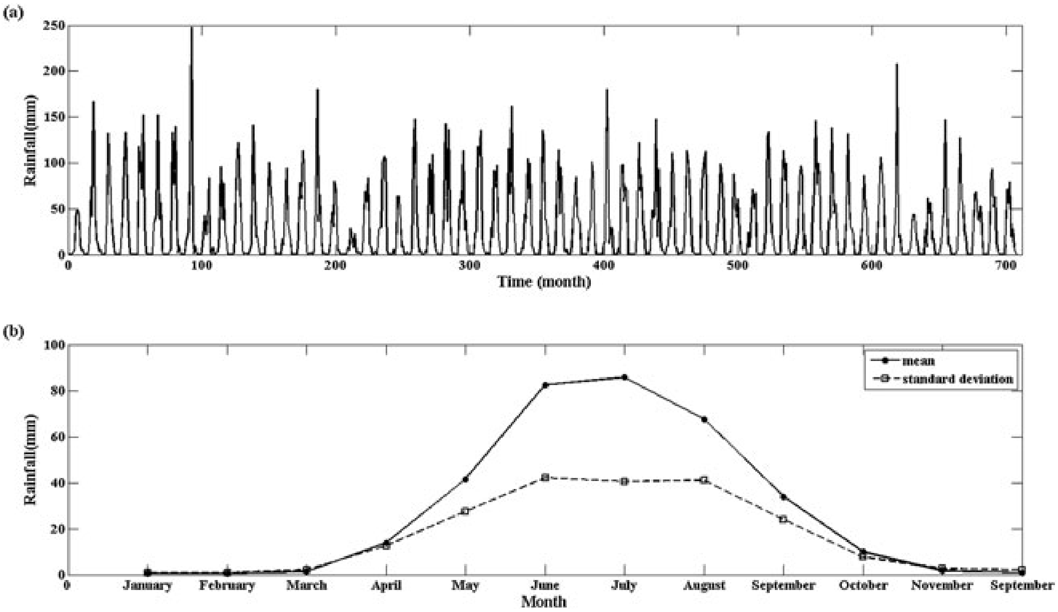

Figure 1. (a) Monthly rainfall time series and (b) monthly distribution of rainfall mean and standard deviation of the Campsie station for 1956–2008

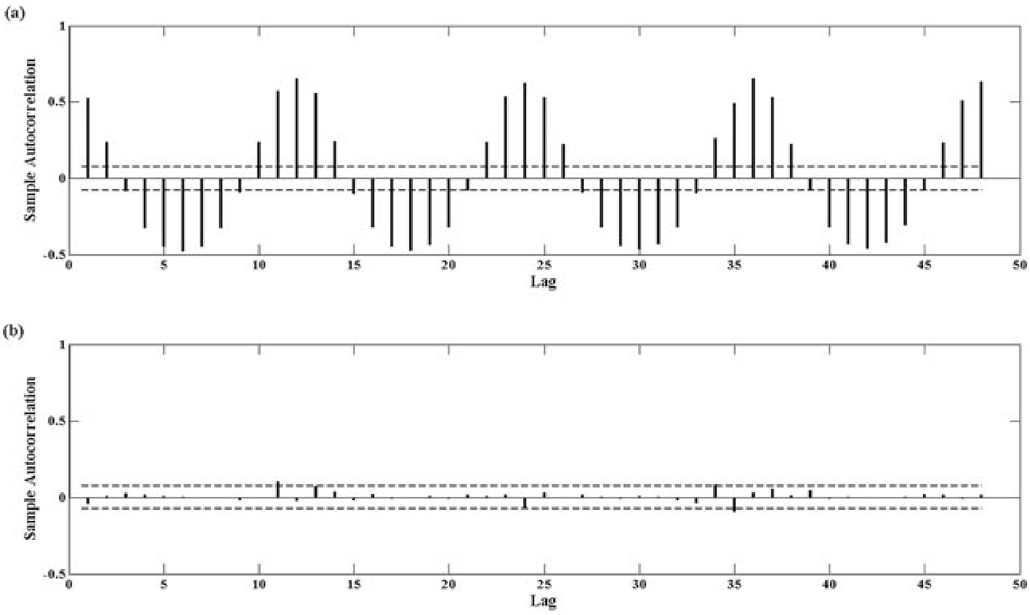

Figure 2. (a) ACF of monthly rainfall time series and (b) ACF of the residuals of the SARIMA$(1,0,1)_{12}$ model for the Campsie station

autocorrelation structure in the residuals, as all coefficients fall within the confidence limit. No seasonality is observed in the residuals, as the coefficients at lags $k = 12, 24, 36, \ldots$ also fall within the confidence interval.

The results of the Ljung–Box test (Figure 3a) also show the adequacy of the model, as the P-values of the Ljung–Box statistic (Q) exceed the critical value ($\alpha = 0.05$). Therefore, the null hypothesis of no autocorrelation structure in the residuals cannot be rejected.

Although the residuals seem statistically uncorrelated according to the ACF and no seasonal structure is observed in the residuals, the ACF of the SSRs is investigated to check the presence of the ARCH effect. The ACF of the SSRs is given in Figure 3b, which illustrates the existence of an ARCH effect in the SSRs of the SARIMA$(1,0,1)_{12}$ model. A seasonal heteroscedasticity is also observed in the SSRs of the SARIMA$(1,0,1)_{12}$ model, as the autocorrelation coefficients exceed the confidence limit at lags $k = 12, 24, 36, \ldots$. The P-values of the Engle's test for the ARCH effect of SSRs of the SARIMA$(1,0,1)_{12}$ model are presented in Figure 3c, which illustrates the presence of heteroscedasticity in the SSRs of the SARIMA$(1,0,1)_{12}$ model. For all lags, the P-values of the test are less than the critical value ($\alpha = 0.05$), and the null hypothesis of no ARCH effect is rejected. In other words, the heteroscedasticity of the residuals of the SARIMA$(1,0,1)_{12}$ model is significant, and we need to apply a GARCH approach to remove it from the residuals of the SARIMA$(1,0,1)_{12}$ model.

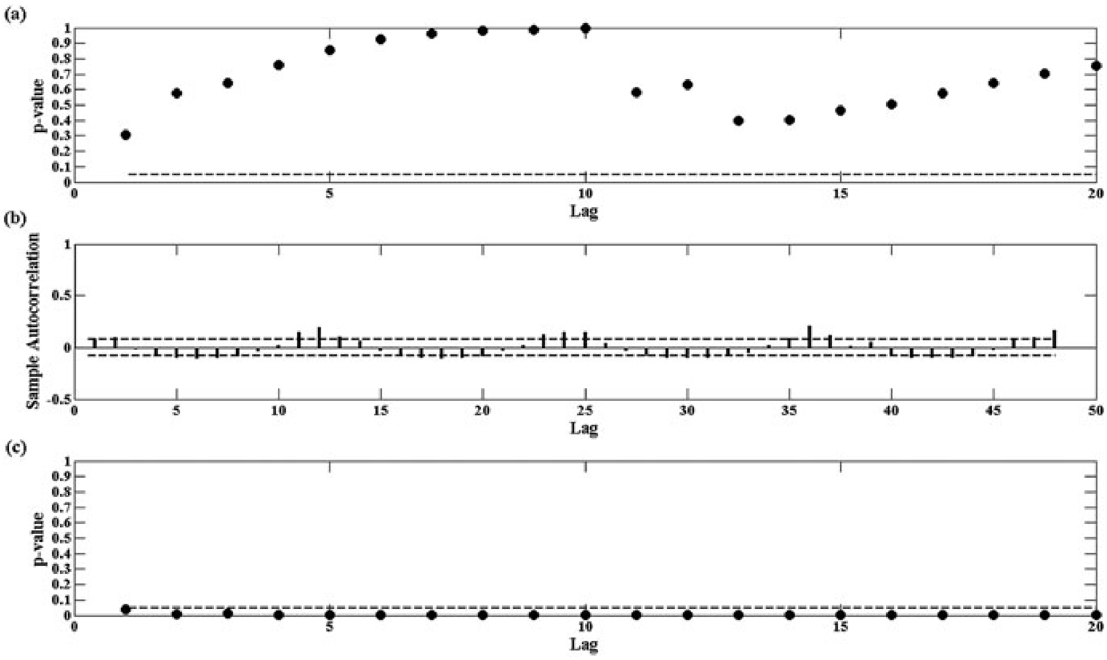

Figure 3. (a) *P*-values of the Ljung–Box test of the residuals, (b) autocorrelation function and (c) *P*-values of the Engle's test for the SSRs of the SARIMA$(1,0,1)_{12}$ model

As seasonality is also observed in the SSRs of the SARIMA$(1,0,1)_{12}$ model, we first try to remove this seasonal variance by adding a seasonal differencing parameter and fitting a SARIMA$(1,1,2)_{12}$ model to the rainfall data. This model includes a seasonal autoregressive parameter, two seasonal moving average parameters and a seasonal differencing operator, $\nabla_S{}^D$.

The ACF of the residuals is first investigated for validating the adequacy of the model. The ACF of the residuals (Figure 4a) indicates the appropriateness of the model, as the autocorrelation coefficients are within the confidence interval of ACF. The Ljung–Box test for the residuals also indicates the adequacy of the SARIMA$(1,1,2)_{12}$ model for the rainfall time series of the Campsie station (Figure 4b). All *P*-values of the test lie outside the critical level ($\alpha = 0.05$) and reject the existence of an autocorrelation structure in the residuals.

For testing the ARCH effect, the Engle's test is used for the SSRs of the SARIMA$(1,1,2)_{12}$ model. The *P*-values of the test (Figure 4c) show that the null hypothesis of no ARCH effect in the SSRs of the SARIMA$(1,1,2)_{12}$ model is rejected, and no improvement of the seasonal heteroscedasticity stabilization is also observed in the SSRs of the SARIMA$(1,1,2)_{12}$ model. Therefore,

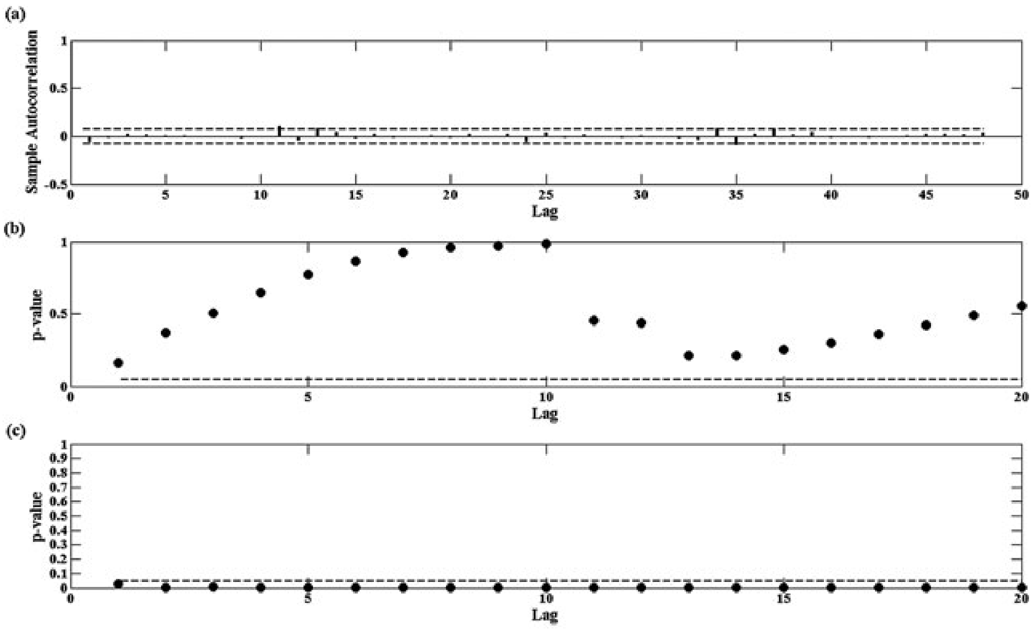

Figure 4. (a) Autocorrelation function of the residuals, (b) *P*-values of the Ljung–Box test of the residuals and (c) *P*-values of the Engle's test for the SSRs of the SARIMA$(1,1,2)_{12}$ model

adding a seasonal differencing parameter cannot remove the seasonal heteroscedasticity, and we try fitting a GARCH model to the residuals of both the SARIMA $(1,0,1)_{12}$ and SARIMA$(1,1,2)_{12}$ models to stabilize the heteroscedasticity.

A GARCH(1,1) model is first fitted to the residuals of the SARIMA$(1,0,1)_{12}$ model. To test the existence of an ARCH effect, the ACF of the SSRs of SARIMA(1,0,1) $_{12}$–GARCH(1,1) (given in Figure 5a) is inspected. The figure indicates some reduction in the autocorrelation structure of the residuals comparing to the ACF of SSRs for the SARIMA$(1,0,1)_{12}$ model given in Figure 3b. For example, the autocorrelations at lags $k=1$ to $k=10$ and $k=13$ to $k=20$ in the ACF of the SSRs of SARIMA(1,0,1) $_{12}$–GARCH(1,1) (Figure 5a) are within the confidence interval whereas these autocorrelations are significant in Figure 3b. However, the seasonality still exists in the SSRs of SARIMA$(1,0,1)_{12}$–GARCH(1,1).

The P-values of the Engle's test for the ARCH effect are also shown in Figure 5b. It can be seen that except for the first four lags ($k=1$–4), all P-values are significantly below the critical value ($\alpha=0.05$), and the null hypothesis of no ARCH effect is rejected. Therefore, the SARIMA $(1,0,1)_{12}$–GARCH(1,1) model is not able to remove the conditional variance, especially the seasonal conditional variance, of the monthly rainfall at the Campsie station.

In the following, a GARCH(1,2) model is fitted to the residuals of the SARIMA$(1,1,2)_{12}$ model. The ACF of the SSRs of the SARIMA$(1,1,2)_{12}$–GARCH(1,2) model is presented in Figure 6a. This figure illustrates no significant autocorrelation structure, as all the coefficients at different lags are within the confidence interval up to lag $k=39$. The

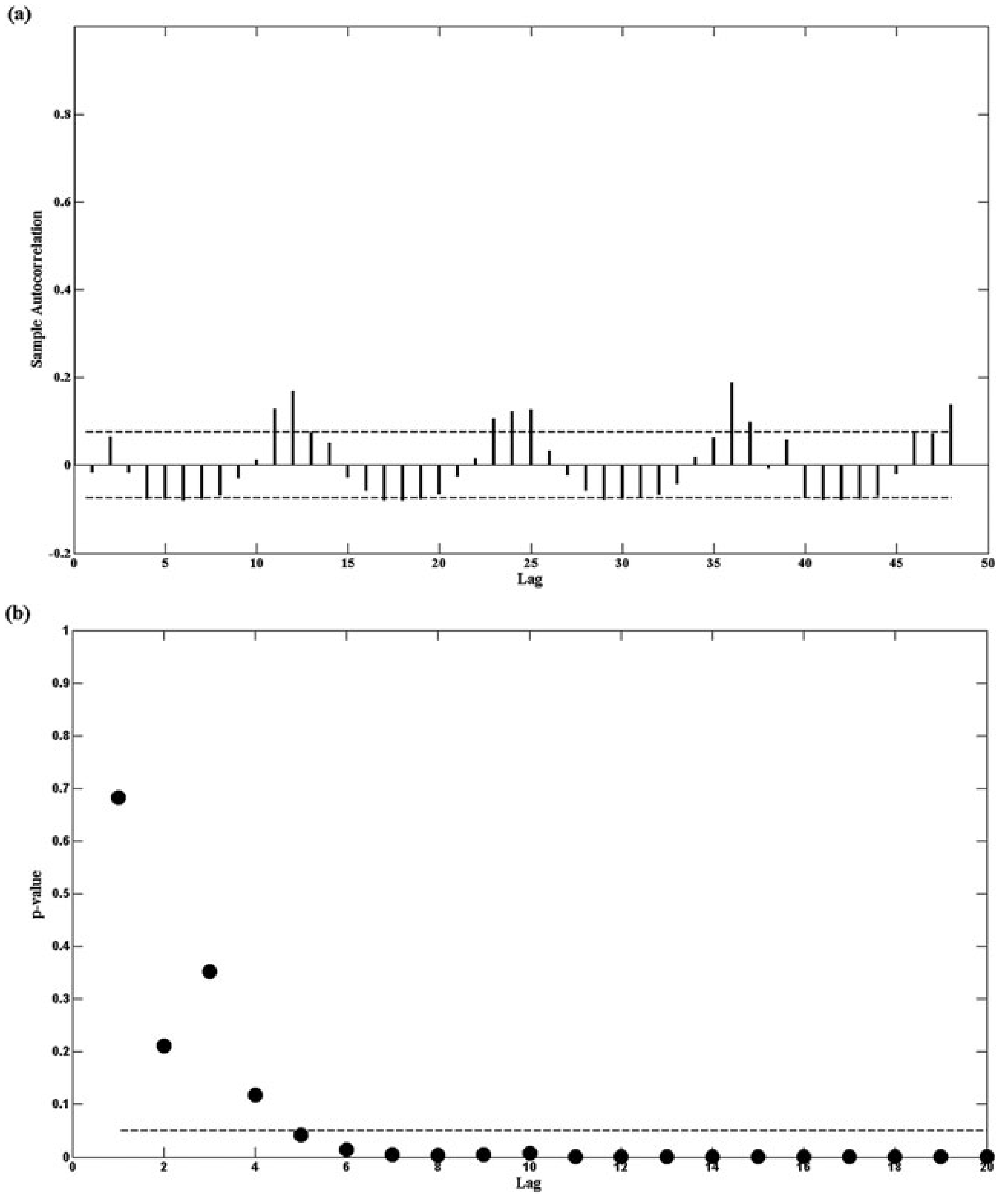

Figure 5. (a) Autocorrelation function and (b) P-values of the Engle's test for the SSRs of the SARIMA$(1,0,1)_{12}$–GARCH(1,1) model for rainfall data of the Campsie station

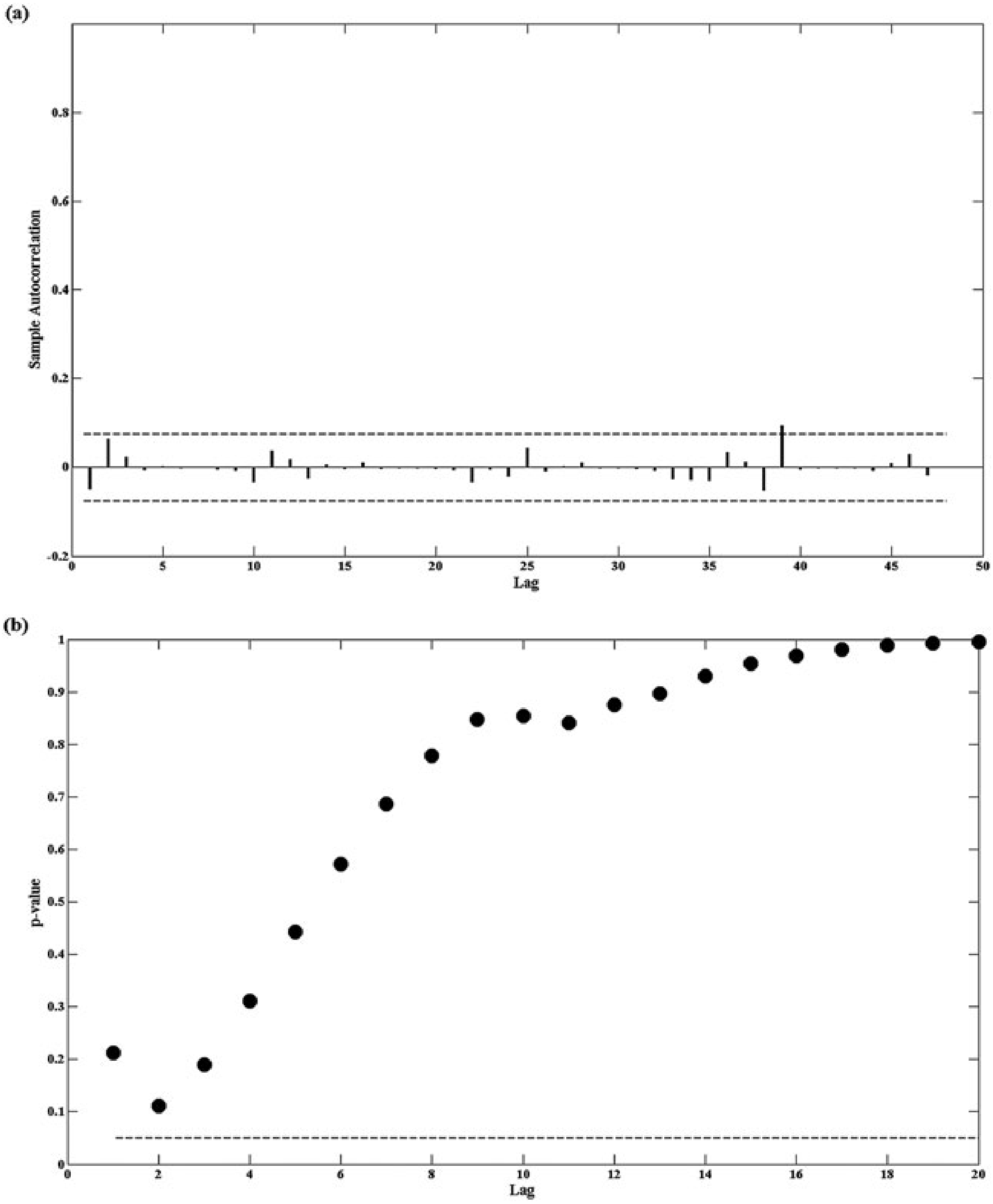

Figure 6. (a) Autocorrelation function and (b) P-values of the Engle's test for the SSRs of the SARIMA$(1,1,2)_{12}$–GARCH(1,2) model for rainfall data of the Campsie station

significant autocorrelation at lag $k = 39$ could be considered as a random effect in the heteroscedasticity of rainfall.

The major feature of Figure 6a is that the seasonal autocorrelation coefficients at lags $k = 12$, 24, 36 and 48 are not significant. This is an important advantage of using a seasonal ARIMA model with an appropriate differencing parameter together with a GARCH approach to remove the seasonal heteroscedasticity from the seasonal hydrologic time series. The P-values of the Engle's test given in Figure 6b also verify the advantage of GARCH modelling approach together with seasonal differencing to stabilize the conditional variance of rainfall time series. All P-values are larger than the critical value ($\alpha = 0.05$), and the null hypothesis of no ARCH effect in the residuals of the SARIMA$(1,1,2)_{12}$–GARCH (1,2) model cannot be rejected.

Results for modelling transformed data. In addition to the investigation of the effect of seasonal differencing on stabilizing the seasonal heteroscedasticity, Box–Cox transformation is also used to stabilize the variance of the rainfall time series. The power parameter of the Box–Cox transformation is $\lambda = 0.112$.

To investigate the effect of seasonal differencing on removing the heteroscedasticity from residuals, two SARIMA models are fitted to the transformed data. In the first model, no seasonal differencing parameter is added whereas for the second model, a seasonal differencing parameter is also included.

The first SARIMA model for Box–Cox transformed rainfall time series of the Campsie station includes two seasonal and two nonseasonal parameters of order 1 to obtain the SARIMA$(1,0,1) \times (1,0,1)$ model. The ACF and the P-values of the Ljung–Box test for the residuals of the model, given in Figure 7a and b, respectively, indicate the acceptance of the null hypothesis of model adequacy. However, if the SSRs of the model are examined for an ARCH effect, a seasonal heteroscedasticity can be observed in the residuals of the SARIMA(1,0,1) × (1,0,1) model (Figure 7c), as the autocorrelation coefficients at lags $k = 12$, 24 and 36 are significant whereas other autocorrelation coefficients are within the confidence level. The P-values of the Engle's test for SSRs are also given in Figure 7d. It can be seen that most of the P-values, but not all of them, are larger than the critical value ($\alpha = 0.05$). Some P-values at lags $k = 12$ to $k = 16$ are less than the critical value and imply the existence of an ARCH effect in the residuals. The Engle's test indicates that the Box–Cox transformation has stabilized the heteroscedasticity of the residuals to a very low level but that some seasonal conditional variances still remain in the residuals. This suggests the need of a GARCH model to stabilize the remaining heteroscedasticity of the SARIMA(1,0,1) × (1,0,1) model fitted to the Box–Cox transformed data. Therefore, we fit a GARCH(1,1) model to the residuals of the SARIMA(1,0,1) × (1,0,1) model. The residuals of the SARIMA(1,0,1) × (1,0,1)–GARCH(1,1) model are then checked for the ARCH effect, inspecting the ACF of SSRs and the P-values of the Engle's test. The results are given in Figure 8. This figure illustrates that the autocorrelation coefficients are within the confidence level of ACF and are not significant. The seasonal autocorrelation coefficients at lags $k = 12$, 24, 36, ... are also insignificant. The result of the Engle's test also indicates no ARCH effect in the SSRs of the GARCH model. Therefore, the SARIMA (1,0,1) × (1,0,1)–GARCH(1,1) model is sufficient for modelling the conditional variance of rainfall time series of the Campsie station.

From the above analysis, a GARCH model seems to be required for modelling the heteroscedasticity in the residuals of the SARIMA(1,0,1) × (1,0,1) model fitted to the Box–Cox transformed rainfall data without using a seasonal differencing parameter. However, the seasonal differencing parameter is also included in the model to check if the seasonal differencing can remove the heteroscedasticity from residuals so that no ARCH effect remains to be modelled using a GARCH approach. Adding a seasonal differencing parameter to the SARIMA(1,0,1) × (1,0,1) model, we fit a SARIMA(1,0,1) × (1,1,1) model to the Box–Cox transformed rainfall time series. The adequacy of the model is confirmed by inspecting the ACF of the residuals and using the Ljung–Box test.

The ACF and the P-values of the Engle's test of SSRs (Figure 9) indicate that if appropriate seasonal differencing is used with Box–Cox transformation, the seasonal heteroscedasticity will be stabilized. Neither nonseasonal nor seasonal heteroscedasticity can be observed in the ACF of the SSRs of the SARIMA (1,0,1) × (1,1,1) model fitted to Box–Cox transformed rainfall data. The P-values of the Engle's test are also larger than the critical value, and no ARCH effect exists in the residuals. Therefore, we agree that an appropriate transformation and deseasonalization approach would result in removing the ARCH effect from the residuals of the SARIMA model for the monthly rainfall data of the Campsie station and that no further GARCH model is necessary for stabilizing the conditional variance of the rainfall data.

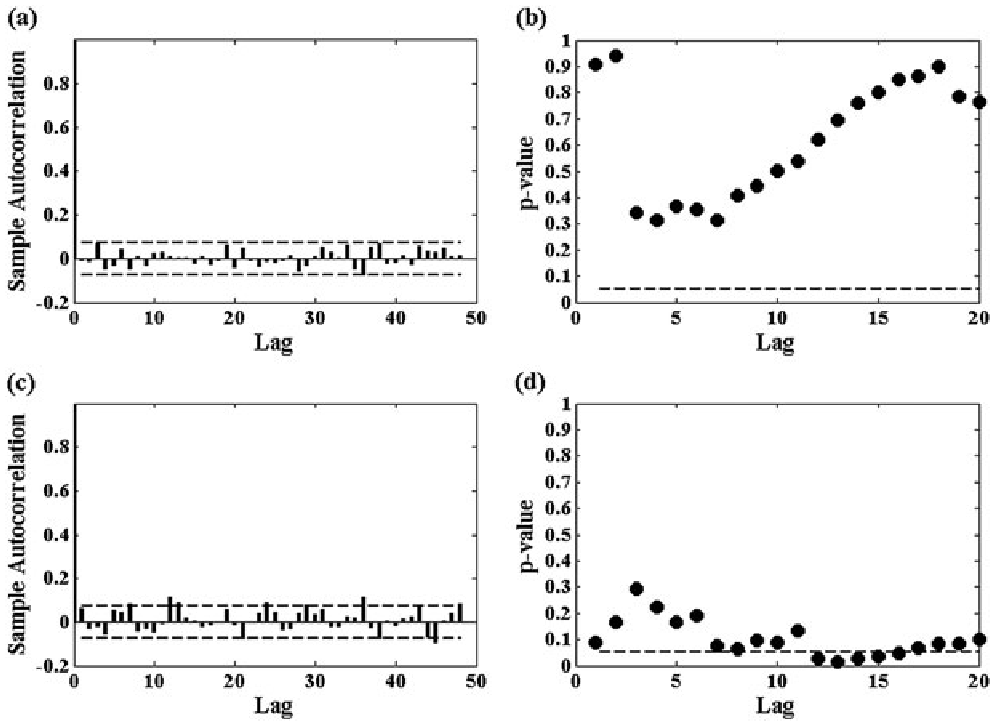

Figure 7. (a) Autocorrelation function of the residuals, (b) P-values of the Ljung–Box test for the residuals, (c) autocorrelation function of the SSRs and (d) P-values of the Engle's test of the SSRs of the SARIMA(1,0,1) × (1,0,1) model for Box–Cox transformed rainfall data of the Campsie station

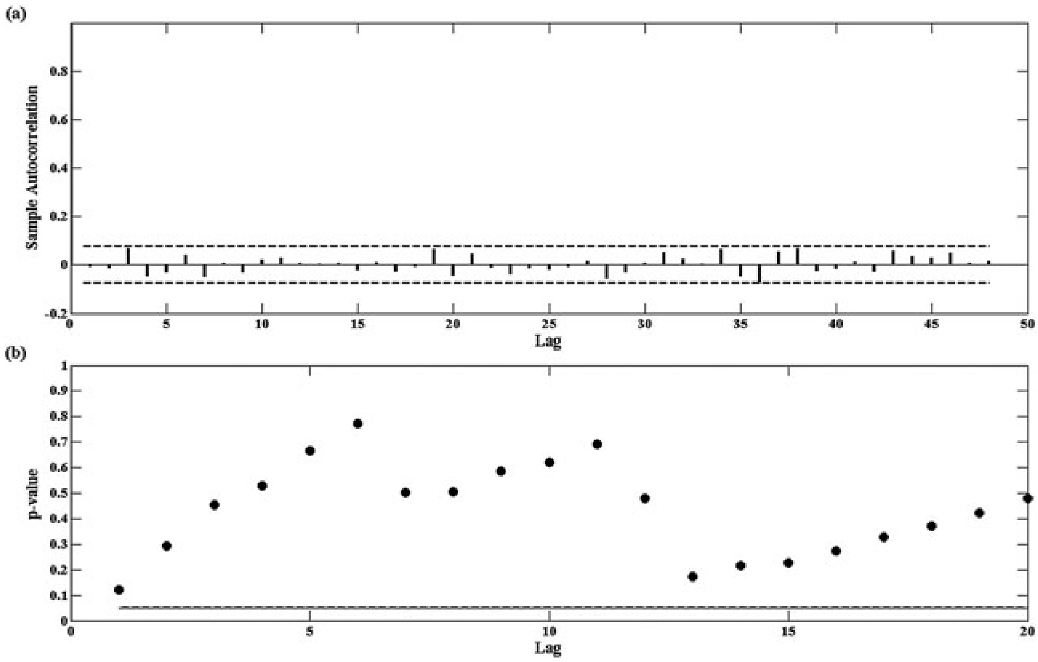

Figure 8. (a) Autocorrelation function and (b) *P*-values of the Engle's test of the SSRs of the SARIMA(1,0,1) × (1,0,1)–GARCH(1,1) model for Box–Cox transformed rainfall data of the Campsie station

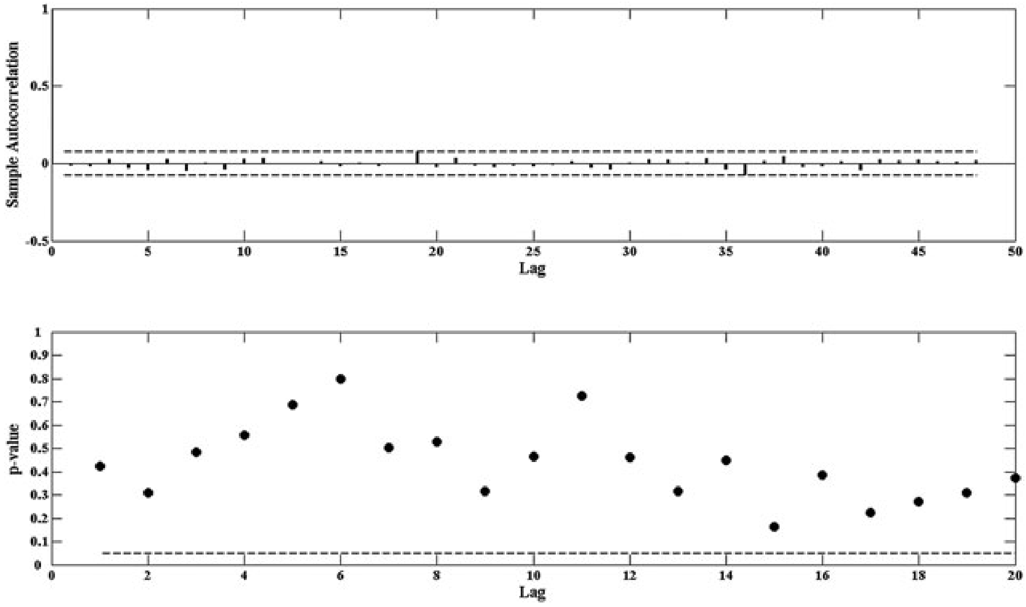

Figure 9. (a) Autocorrelation function and (b) *P*-values of Engle's test of the SSRs of the SARIMA(1,0,1) × (1,1,1) model for Box–Cox transformed rainfall data of the Campsie station

Rainfall for an arid region

Data. The second case study deals with modelling the monthly rainfall time series of the Isfahan station (32°37'N and 51°40'E), Isfahan province (elevation: 1550.4 m), Iran. The station is located in the semi-arid region of Iran with a mean annual rainfall of 122.8 mm for the period 1951–2005. The monthly rainfall time series of the Isfahan station and the monthly distribution of the rainfall mean and standard deviation are given in Figure 10a and b, respectively.

In the following two sections, we present the results of time series modelling for the original and Box–Cox transformed rainfall data of the Isfahan station to discuss the ARCH effect of the rainfall in an arid region.

Results for modelling original data. The ACF of the rainfall (Figure 11a) indicates the seasonal behaviour of rainfall at the Isfahan station. Therefore, the SARIMA (0,0,1) × (1,0,1) model is first fitted to rainfall data. This model has three parameters: one nonseasonal moving average parameter and two seasonal autoregressive and seasonal moving average parameters, all of order 1. The ACF of the residuals of this model (Figure 11b) and the *P*-values of Ljung–Box test (Figure 11c) indicate the adequacy of the

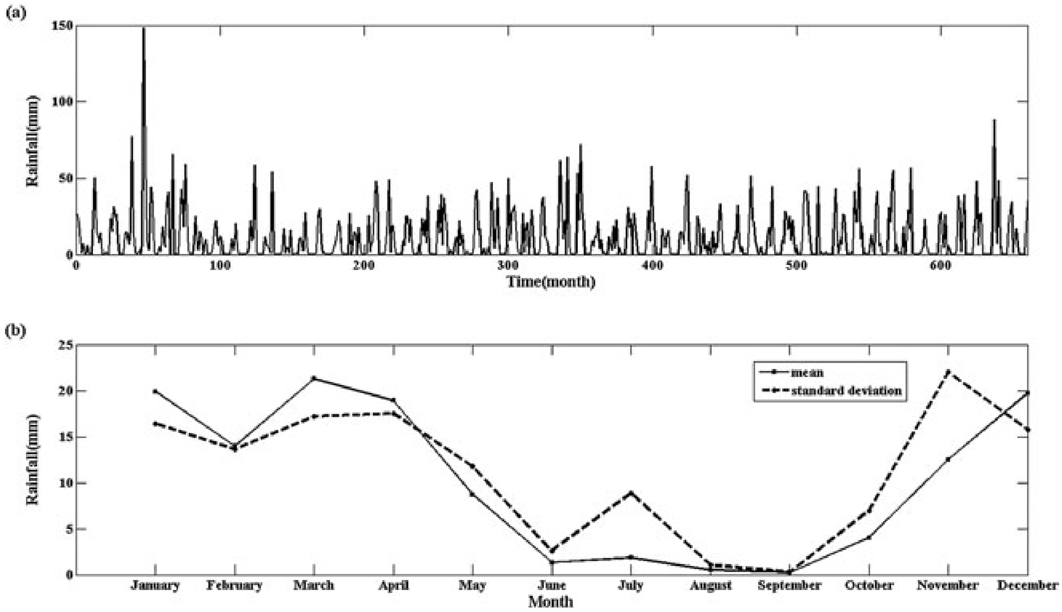

Figure 10. (a) Monthly rainfall time series and (b) monthly distribution of rainfall mean and standard deviation of the Isfahan station for 1951–2005

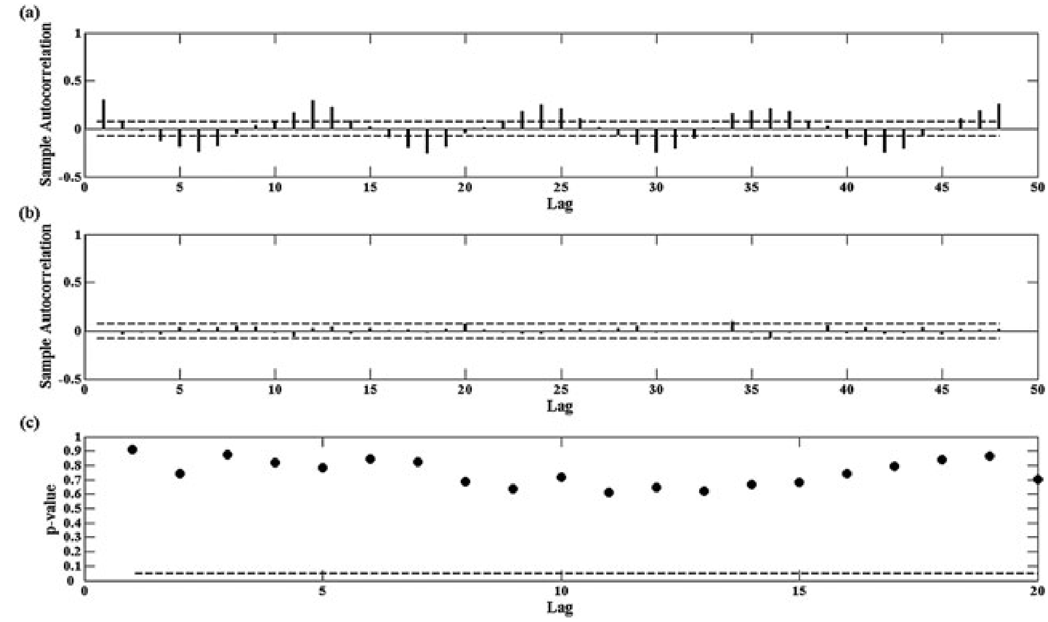

Figure 11. (a) ACF of the Isfahan rainfall time series, (b) ACF of the residuals and (c) P-values of the Ljung–Box test of the SARIMA(0,0,1) $\times$ (1,0,1) model for the Isfahan station

model. The existence of an ARCH effect in the SSRs of the above model is investigated by inspecting the ACF of SSRs and using the Engle's test. The ACF (Figure 12a) indicates neither nonseasonal nor seasonal heteroscedasticity in the residuals, as the autocorrelation coefficients are within the confidence limits. However, at lags $k = 8$ and $k = 20$, two autocorrelation coefficients fall outside the confidence level. The P-values of the Engle's test (Figure 12b) prove no ARCH effect in the SSRs of the SARIMA(0,0,1) $\times$ (1,0,1) model and suggest that the significant autocorrelation coefficients at lags $k = 8$ and $k = 20$ could be considered as random effects on the conditional variance of rainfall at the Isfahan station.

Adding seasonal differencing to the above model to obtain a SARIMA(0,0,1) $\times$ (1,1,1) model shows the same adequacy of the model according to the ACF and P-values of the Ljung–Box test and the same ARCH structure in the residuals as those for the SARIMA (0,0,1) $\times$ (1,0,1) model. No seasonal heteroscedasticity is observed, and the autocorrelation coefficients are still significant at lags $k = 8$ and $k = 20$.

The significant autocorrelation coefficients at lags $k = 8$ and $k = 20$ could be considered as random conditional variances of rainfall at the Isfahan station. These random effects could be related to an uncertainty in climate conditions in the arid regions. The timing and amount of

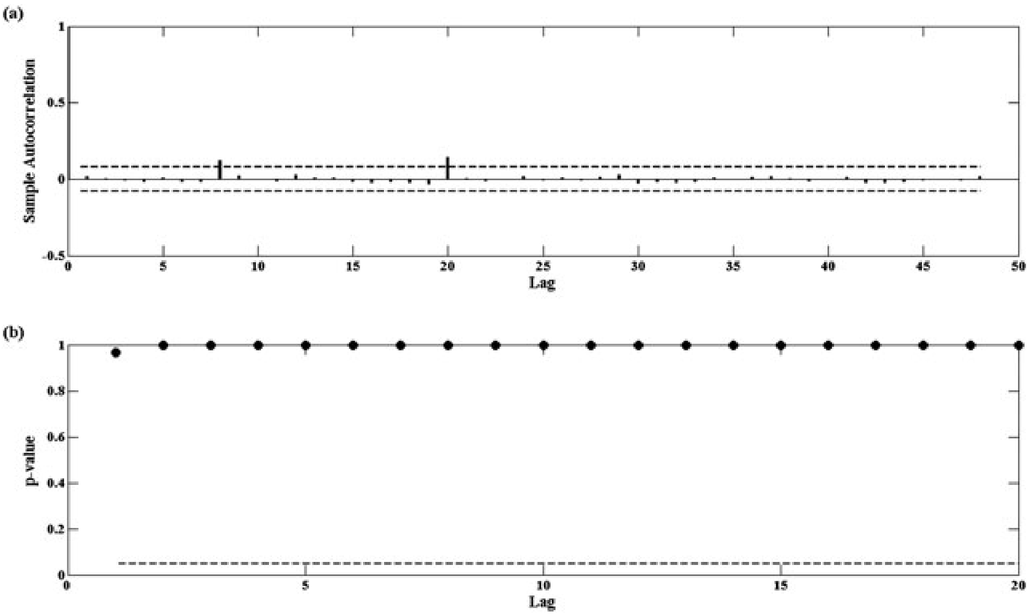

Figure 12. (a) Autocorrelation function and (b) *P*-values of the Engle's test of the SSRs of the SARIMA(0,0,1) × (1,0,1) model for the Isfahan station

precipitation indicate a high irregular fluctuation in the arid regions and may impose random effects on the conditional variance of rainfall.

One can accept that SARIMA(0,0,1) × (1,1,1) is enough for modelling the rainfall time series of the Isfahan station at this level considering the lags $k = 8$ and $k = 20$ as random ARCH effects and does not feel necessary to use a GARCH model to remove these random effects. Nevertheless, we continue our modelling approach by fitting a GARCH model to the residuals of the SARIMA model in order to investigate the advantage/disadvantage of the GARCH models.

Trying to remove the random ARCH effect from the residuals by fitting a GARCH model, we use SARIMA (0,0,1) × (1,0,1)–GARCH(0,2) and SARIMA (0,0,1) × (1,1,1)– GARCH(0,2). As the ACF and the Engle's test for the residuals of both the above models are the same, we illustrate the ACF of the SSRs and the *P*-values of the Engle's test for the SARIMA (0,0,1) × (1,1,1)–GARCH(0,2) model in Figure 13a and b, respectively. No ARCH effect is observed in the residuals of the SARIMA(0,0,1) × (1,1,1)–GARCH (0,2) model, as all autocorrelation coefficients fall within the confidence interval of the ACF and all *P*-values are larger than the critical value. It is important to note that neither nonseasonal nor seasonal heteroscedasticity is observed in the residuals of the GARCH model and that the random conditional variances at lags $k = 8$ and $k = 20$ have also been removed from the residuals. This suggests the appropriateness of a GARCH approach for modelling the

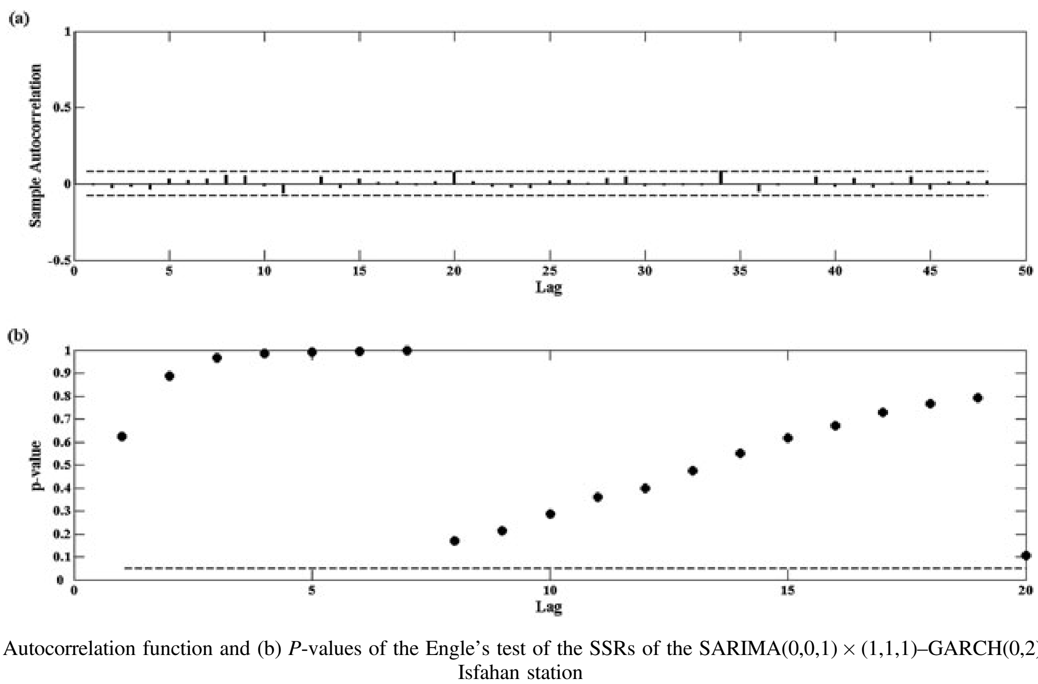

Figure 13. (a) Autocorrelation function and (b) *P*-values of the Engle's test of the SSRs of the SARIMA(0,0,1) × (1,1,1)–GARCH(0,2) model for the Isfahan station

heteroscedasticity of rainfall time series in an arid region where rainfall indicates irregular temporal fluctuations and variation.

Results for modelling transformed data. Time series modelling of the original rainfall data of Isfahan indicates the advantage of using a GARCH model to remove the conditional variance of the residuals of a SARIMA model. However, we also apply a transformation method to check if it will reduce the heteroscedasticity of rainfall data so that no ARCH effect remains to be modelled by a GARCH approach.

The Box–Cox transformation method with $\lambda = 0.056$ is applied to transform the rainfall data of the Isfahan station. Two SARIMA models with and without a differencing parameter are then fitted to the transformed data in order to see the adequacy of Box–Cox transformation in removing heteroscedasticity from the residuals of the SARIMA models.

The SARIMA model fitted to the transformed data without applying seasonal differencing is SARIMA $(0,0,2) \times (1,0,2)$. The model is adequate according to the ACF (Figure 14a) and *P*-values of Ljung–Box (Figure 14b) of the residuals.

The heteroscedasticity remaining in the residuals of the above model is investigated by checking the ACF of SSRs (Figure 15a) and using the Engle's test for an ARCH effect (Figure 15b). The ACF of the SSRs indicates the existence

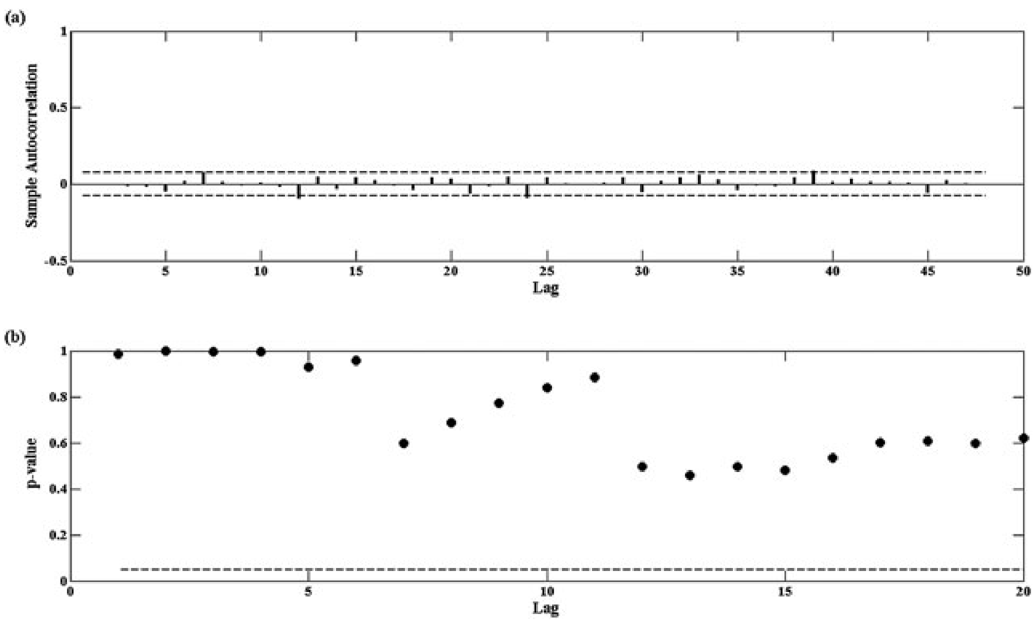

Figure 14. (a) ACF of the residuals and (b) *P*-values of the Ljung–Box test of the SARIMA $(0,0,2) \times (1,0,2)$ model for Box–Cox transformed rainfall data of the Isfahan station

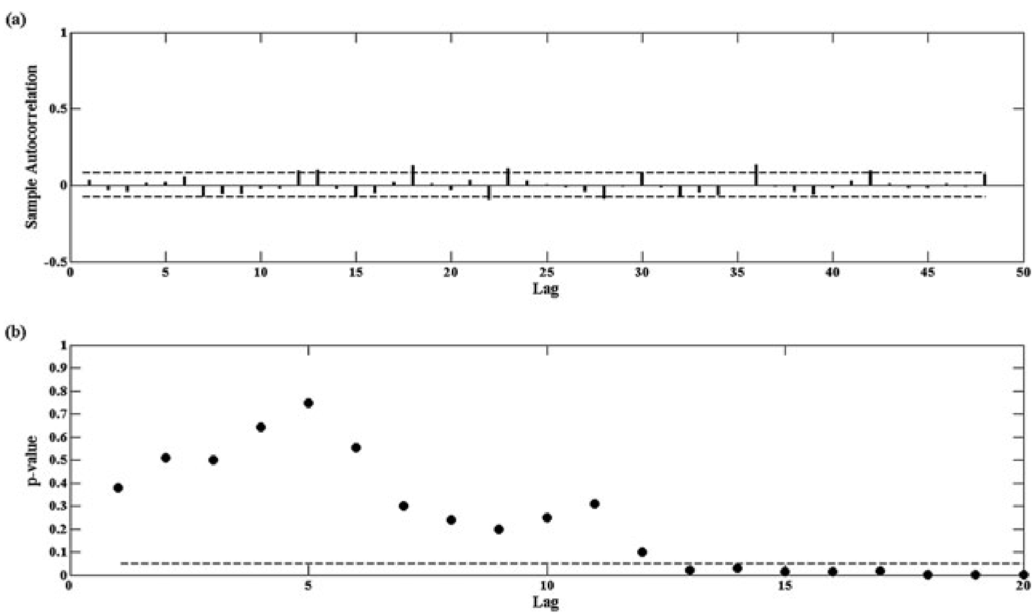

Figure 15. (a) Autocorrelation function and (b) *P*-values of the Engle's test of the SSRs of the SARIMA$(0,0,2) \times (1,0,2)$ model for Box–Cox transformed rainfall data of the Isfahan station

of both nonseasonal and seasonal heteroscedasticity. The results of the Engle's test also indicate an ARCH effect in the residuals of the SARIMA(0,0,2) × (1,0,2) model fitted to the Box–Cox transformed data.

By adding a seasonal differencing operator to the above model, the effect of seasonal differencing on stabilizing the seasonal heteroscedasticity is examined. The SARIMA(0,0,2) × (1,1,2) model is also adequate according to the Ljung–Box test for the residuals. However, both seasonal heteroscedasticity and nonseasonal heteroscedasticity are still observed in the SSRs (Figure 16a). The Engle's test indicates some improvement of modelling heteroscedasticity of the residuals with the addition of a seasonal differencing parameter to the model inasmuch as some of the P-values are larger than the critical value ($\alpha = 0.05$) (Figure 16b). However, the ARCH effect still remains in the residuals, and we need to apply a GARCH approach to model this heteroscedasticity in the residuals.

The GARCH model is then fitted to the residuals to obtain a SARIMA(0,0,2) × (1,0,2)–GARCH(2,3) model. To check the ARCH effect of the residuals, the ACF of SSRs and the P-values of the Engle's test are given in Figure 17. The ACF of SSRs (Figure 17a) indicates the adequacy of the GARCH approach for modelling the heteroscedasticity of rainfall time series. However, a very weak seasonal heteroscedasticity is observed as the seasonal

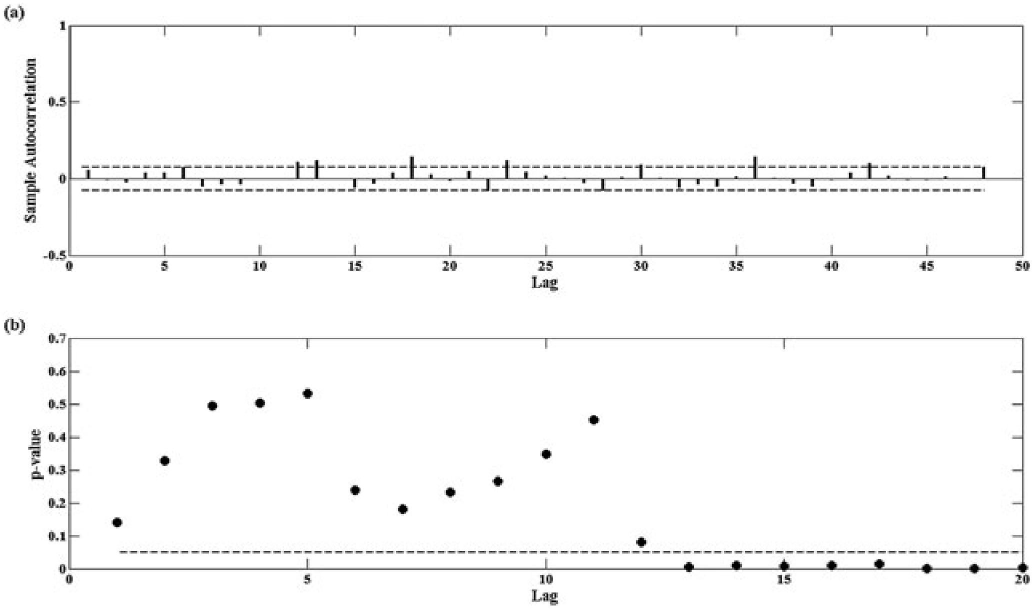

Figure 16. (a) Autocorrelation function and (b) P-values of the Engle's test of the SSRs of the SARIMA(0,0,2) × (1,1,2) model for Box–Cox transformed rainfall data of the Isfahan station

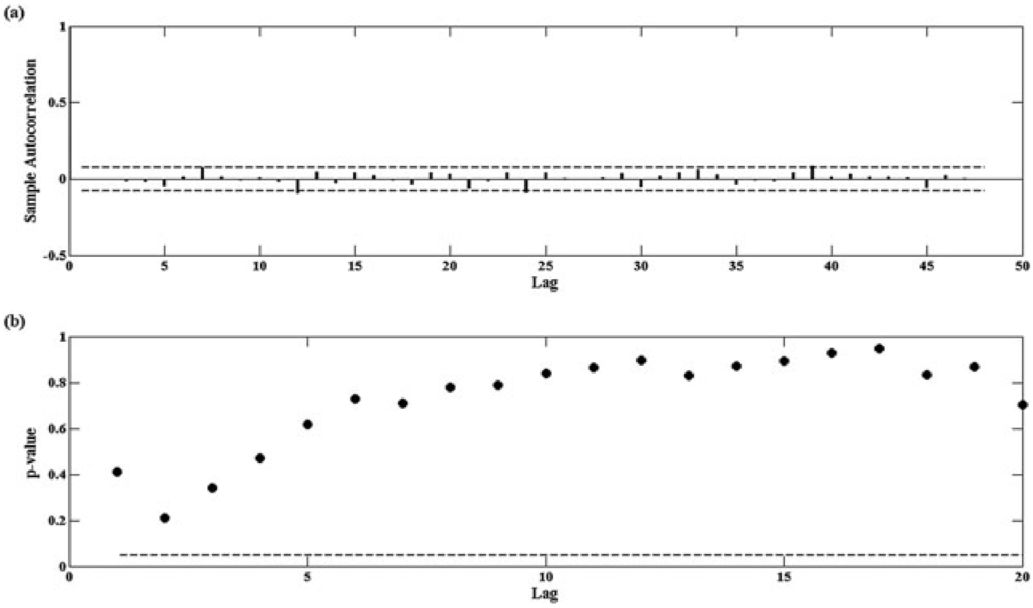

Figure 17. (a) Autocorrelation function and (b) P-values of the Engle's test of the SSRs of the SARIMA(0,0,2) × (1,0,2)–GARCH(2,3) model for Box–Cox transformed rainfall data of the Isfahan station

autocorrelation coefficients at lags $k = 12$ and $k = 24$ fall outside the confidence level. If we fit a GARCH model to the residuals of the SARIMA(0,0,2) × (1,1,2) model to obtain a SARIMA(0,0,2) × (1,1,2)–GARCH(2,2) model, the evident of the weak heteroscedasticity is also removed. The ACF and the *P*-values of the Engle's test for the above model are given in Figure 18a and b, respectively. It is clear that the GARCH model has removed both seasonal and nonseasonal heteroscedasticity from the residuals.

Modelling the transformed data suggests that Box–Cox transformation and seasonal differencing cannot remove conditional variance from the residuals of the Isfahan station without applying a GARCH model.

MODEL COMPARISON

This section provides the criteria for model performance evaluation and cross-comparison of the SARIMA and GARCH models fitted to rainfall data at the Campsie and Isfahan stations.

Firstly, the criteria for the Campsie station are considered (Table I). For the original rainfall data, the SARIMA model with a seasonal differencing parameter, SARIMA$(1,1,2)_{12}$, indicates a slightly better performance than other models according to RMSE, AME and PDIFF. It suggests that SARIMA$(1,1,2)_{12}$ is a better model for predicting peak rainfall in the Campsie station than other models. Although the performances of the SARIMA and GARCH models look almost the same based on the error and dimensionless metrics, the Diebold–Mariano (DM) statistic indicates a significant difference between SARIMA and SARIMA–GARCH's performances with and without seasonal differencing. Nevertheless, the model with a seasonal differencing parameter indicates more negative DM statistics compared with the model without differencing.

On the other hand, for Box–Cox transformed data, the SARIMA and GARCH models seem to relatively outperform the SARIMA and GARCH models fitted to the original rainfall data according to dimensionless metrics. However, no improvement in model accuracy is observed comparing the GARCH model with the SARIMA model for the Box–Cox transformed rainfall time series of the Campsie station according to DM statistics. This suggests that the application of a GARCH model for the rainfall data of the Campsie station, as a sample of a humid region, does not necessarily improve the performance of a SARIMA model after Box–Cox transformation.

Looking at the criteria for the Isfahan station in Table II also suggests that a GARCH model does not improve the performance of the SARIMA model fitted to the original rainfall time series.

However, for Box–Cox transformed rainfall time series, the GARCH models perform better than the SARIMA models. Both global metrics, R^2 and RMSE, show the best performance for the SARIMA(0,0,2) × (1,1,2)–GARCH (2,2) model. Based on other error metrics, the GARCH model also gives the lowest error. The dimensionless metrics also suggest a better agreement between the GARCH modelled and observed rainfall than that of the SARIMA model. However, the null hypothesis of equal accuracy is rejected for all SARIMA–GARCH against SARIMA models. This suggests that there is no remarkable difference in the performance of the SARIMA–GARCH model against a SARIMA model for the Isfahan rainfall time series. It is also important to note that the performance of the time series models fitted to transformed rainfall data, both in arid and humid regions, is much better than the models fitted to the original data, especially for the rainfall data of the Isfahan station, for which the performance of the model is doubled after Box–Cox transformation according to R^2 and CE.

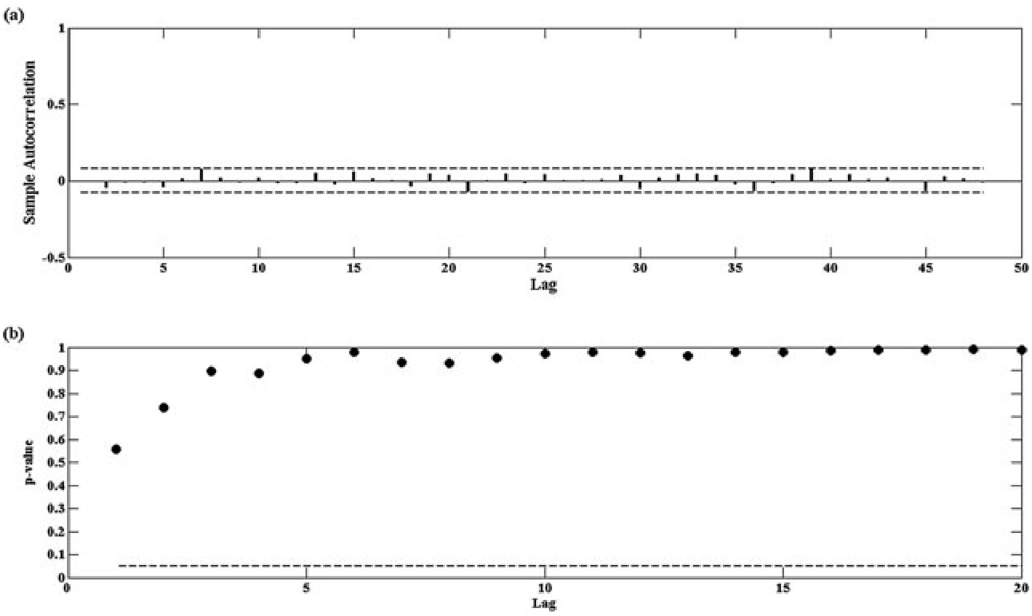

Figure 18. (a) Autocorrelation function and (b) *P*-values of the Engle's test of the SSRs of the SARIMA(0,0,2) × (1,1,2)–GARCH(2,2) model for Box–Cox transformed rainfall data of the Isfahan station

Table I. Model criteria for the Campsie rainfall time series

Variable	Model	AME	PDIFF	MAE	RMSE	RAE	MRE	R^2	CE	IoAd	DM
Original rainfall time series	SARIMA$(1,0,1)_{12}$	181.18	156.12	13.36	23.35	0.43	−1.76	0.65	0.65	0.88	−14.7*
	SARIMA$(1,0,1)_{12}$–GARCH(1,1)	181.79	157.51	13.23	23.36	0.42	−1.22	0.65	0.65	0.88	
	SARIMA$(1,1,2)_{12}$	174.77	156.01	13.30	23.33	0.42	−1.70	0.66	0.66	0.89	−18.01*
	SARIMA$(1,1,2)_{12}$–GARCH(1,2)	182.48	162.53	13.18	23.36	0.42	−0.71	0.66	0.64	0.87	
Box–Cox transformed rainfall	SARIMA(1,0,1) × (1,0,1)	0.43	0.20	0.09	0.12	0.33	−0.01	0.85	0.85	0.95	371.1
	SARIMA(1,0,1) × (1,0,1)–GARCH(1,1)	0.44	0.19	0.09	0.12	0.32	−0.01	0.85	0.85	0.96	
	SARIMA(1,0,1) × (1,1,1)	0.43	0.19	0.09	0.12	0.32	0.0	0.86	0.86	0.96	–

*Significant at 5% level or better.

Table II. Model criteria for the Isfahan rainfall time series

Variable	Model	AME	PDIFF	MAE	RMSE	RAE	MRE	R^2	CE	IoAd	DM
Original rainfall	SARIMA(0,0,1) × (1,0,1)	134.13	107.80	7.85	13.03	0.70	−7.01	0.27	0.27	0.64	0.05
	SARIMA(0,0,1) × (1,0,1)–GARCH(0,2)	133.16	109.24	7.98	12.94	0.72	−7.07	0.28	0.28	0.65	
	SARIMA(0,0,1) × (1,1,1)	134.43	109.49	8.01	13.15	0.71	−7.38	0.27	0.27	0.64	0.12
	SARIMA(0,0,1) × (1,1,1)–GARCH(0,2)	134.78	110.52	8.05	13.14	0.72	−7.44	0.27	0.27	0.64	
Box–Cox transformed rainfall	SARIMA(0,0,2) × (1,0,2)	5.49	1.94	1.04	1.40	0.53	0.30	0.55	0.55	0.84	−0.07
	SARIMA(0,0,2) × (1,0,2)–GARCH(2,3)	5.38	1.83	1.04	1.39	0.53	0.31	0.56	0.55	0.85	
	SARIMA(0,0,2) × (1,1,2)	5.25	1.85	1.02	1.39	0.51	0.32	0.57	0.57	0.85	0.15
	SARIMA(0,0,2) × (1,1,2)–GARCH(2,2)	5.19	1.62	1.00	1.36	0.51	0.31	0.58	0.58	0.86	

Nevertheless, the DM test results indicate that the null hypothesis of equal modelling accuracy is only rejected for SARIMA–GARCH models for original rainfall time series at the Campsie station. Therefore, one can see that the main advantage of a GARCH model is the ability to capture the heteroscedasticity of the residuals of a SARIMA model (as was also reported by Romilly (2005) for global mean temperature time series modelling) and there is no guarantee of a better performance by a GARCH model, at least for our case study.

DISCUSSION

The volatility, the time-dependent variance or the conditional heteroscedasticity, is usually neglected in hydrologic time series modelling. The present study shows that although the seasonal ARIMA model seems to be adequate for modelling the conditional mean of the monthly rainfall time series and its seasonal variation, a seasonal ARCH effect remains in the residuals. Fitting a SARIMA model to the sample rainfall data set used in this study, we noticed the inadequacy of SARIMA models for capturing the seasonal conditional variance of monthly rainfall for both sample data sets coming from the humid and arid regions even when adding a seasonal differencing parameter to the model.

Adding a seasonal differencing parameter to a SARIMA model does not remove the seasonal heteroscedasticity from the residuals of the model. This phenomenon is observed for rainfall data from both humid and arid regions. However, the Box–Cox transformation together with seasonal differencing seemed to be an efficient approach for capturing the seasonal heteroscedasticity of rainfall data of the humid region (the Campsie station) but not a sufficient method for the rainfall data from the arid region (the Isfahan station). However, these results are explanatory, and future analyses are necessary to generalize them.

This suggests that there may be a very irregular conditional variance in rainfall data in arid regions that cannot be removed by seasonal differencing and Box–Cox transformation. The remaining of an ARCH effect in the residuals of the SARIMA model for Box–Cox transformed rainfall data in the arid region may be due to the perturbations of temperature and evapotranspiration fluctuations and nonequilibrium interaction of the earth and atmospheric components in the arid regions. The reasons for the irregular ARCH effect in the rainfall data of the Isfahan station and its relationship to (local or regional) climate fluctuations should be carefully examined using a larger database in different arid regions of the world.

The investigation of the causes of the seasonal ARCH effect and the causes of the inadequacy of the linear models commonly used for seasonal hydrologic time series modelling is out of the scope of the present study. However, it can be assumed that the seasonal heteroscedasticity may be the result of the seasonality of the atmospheric and climatic factors and components that influence the seasonal variance of hydrological variables. The irregular pattern of the climate fluctuations in the arid regions may result in an irregular conditional variance of rainfall in these regions as illustrated by the Isfahan rainfall case study. The above statement on irregular conditional variance requires a more careful investigation, which may be the topic of future studies.

Although the GARCH model indicates the capability of modelling conditional variance, it does not improve the efficiency of the SARIMA models, especially for rainfall data in the arid region. However, the performance of both the SARIMA and GARCH models fitted to transformed rainfall data in the arid region is twice better than that of the models fitted to the original rainfall data according to multi-criteria error evaluation. This can be an important aspect of the rainfall time series modelling in the arid regions that should be carefully considered. Box–Cox transformation seems to be an effective method to reduce the (hidden) variance of rainfall in the arid regions, which also improves the performance of time series models.

It should also be mentioned that the DM test did not show a notable difference between SARIMA and SARIMA–GARCH prediction performances, except for the nontransformed rainfall time series from the humid region.

The results of the present study suggest that using the GARCH approach together with an appropriate transformation technique may increase the performance of the time series models in some cases, such as our example of the Campsie station, and may stabilize the heteroscedasticity of rainfall time series. However, from a parsimonious point of view and as a disadvantage of the GARCH modelling approach, it seems that adding more parameters into rainfall time series models by a GARCH model may not guarantee achieving better rainfall prediction accuracy.

CONCLUSIONS AND FUTURE WORK

This study illustrated the GARCH modelling approach for a rainfall time series with seasonal variation and showed the advantage of a GARCH approach to model the conditional variance of rainfall data. In order to better understand the advantage/disadvantage of the GARCH modelling approach for hydrologic time series modelling, especially for rainfall in arid regions, the use of a larger database from different regions of the world is strongly recommended in future studies. Simulation-based studies are also necessary to confirm and generalize the results of the present study. Space limitations prevented this from being done in the current study, and this should be the topic of future research efforts. In addition, the advantage/disadvantage of GARCH models over SARIMA models for out-of-sample forecasting would be important to consider in future investigations.

An interesting topic for future GARCH modelling efforts deals with modelling nonstationary hydrological time series through climate change conditions, especially time series with change in higher order moments such as the variance, skewness and kurtosis. The application of multivariate GARCH models in order to investigate the shift of the conditional variance from one variable such as rainfall into another variable such as streamflow is also an interesting topic for future works. Using a combination of GARCH models with other time series modelling approaches, such as fractional and periodic time series models, would be useful for modelling persistence and periodicity in the variance of different hydrologic and climatic variables. Coupling GARCH models with smoothing approaches and nonparametric methods would also be helpful in modelling conditional variance of the hydrologic time series.

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