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Assessing Multi-site Drought Connections in Iran Using Empirical Copula

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Abstract Drought is a multi-dimensional natural hazard with stochastic characteristics usually related to each other. Separate univariate statistical models cannot capture the important relationships among drought characteristics, that is, severity and duration. In this study, an empirical copula is employed to construct a bivariate model of droughts, where droughts are defined as continuously negative standardized precipitation index (SPI) periods with one SPI value reaching −1 or less. Bivariate frequency analyses in terms of recurrence intervals are performed using the established empirical copula-based bivariate drought model. The inter-connection among different regions of droughts is explored by a lower tail dependence coefficient. A nonparametric estimation based on an empirical copula is employed pairwisely to calculate the lower tail dependence coefficient among stations. The proposed method is applied to six rainfall gauge stations in Iran to explore drought properties of single sites as well as the inter-connection among multi-sites. The results show that greater mean drought severity and duration are associated with the least arrival rate of drought events, which occurs at the Ahwaz station. The tail dependence analysis reveals that distance between stations is not a key parameter. Generally, the Ahwaz and Isfahan stations have the highest probability of simultaneous droughts among the six stations.

Keywords Drought · Standardized precipitation index (SPI) · Empirical copula · Tail dependence coefficient · Iran

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1 Introduction

Drought is one of the most complex and harmful natural phenomenon which originates from deficiency in precipitation. Prolonged and severe droughts may induce significant water supply problems for domestic and agricultural requirements. Wilhite [40] has indicated that droughts cause an average of $6–8 billion in global damages every year and affect more people than any other natural disaster. In order to monitor droughts and implement drought mitigation measures timely to reduce potential economic losses, several drought indices using diverse variables for drought quantification have been developed during the last century [11].

Rainfall data are widely used to calculate drought indices because long-term rainfall data are more readily available than other hydrologic variables. The standardized precipitation index (SPI), developed by McKee et al. [18], has been proposed for detection of droughts using precipitation data at different time scales. Given its simplicity in calculation and independence of geographical and topographical characteristics, the SPI has become a popular tool for drought analyses worldwide (e.g., [1, 17, 22, 39, 41]).

Recognizing drought characteristics as random variables, a considerable amount of probabilistic models have been developed to investigate droughts. Most drought studies treat drought characteristics such as severity and duration separately and construct a probabilistic model for each drought variable. The relationships among correlated drought characteristics are ignored by these models. Some bivariate drought models have been proposed recently, e.g., Shiau and Shen [36], González and Valdés [8], Bonaccorso et al. [2], Kim et al. [16], and Salas et al. [25], to consider simultaneously two correlated drought characteristics, that is, drought severity and duration.

Since the correlated drought characteristics may have different marginals, copulas offer a flexible way to construct such multivariate models. Shiau [33] and Shiau et al. [34] explore bivariate drought frequency using copulas. Shiau and Modarres [35] extend the bivariate drought model to construct copula-based drought severity–duration–frequency curves. Serinaldi et al. [32] use copulas to develop the four-dimensional joint distribution for the drought properties: drought length, mean and minimum SPI values, and drought mean areal extent. Kao and Govindaraju [15] develop a copula-based joint deficit index simultaneously considering precipitation and streamflow to monitor drought status. The flexibility offered by copulas to construct correlated multivariate models leads to increasing applications of copulas to model complex hydrological processes (e.g., [4–7, 9, 13, 14, 23, 24, 26–28, 37, 42–44]).

This study extends previous studies of the copula-based single-site drought model to explore spatial relationships among multiple sites located in different climate regions. Instead of using theoretical copula functions, an empirical copula is used to construct a bivariate drought model for various time-scale precipitation data. The inter-connections of droughts among multi-sites are evaluated by the tail dependence coefficient because droughts are considered as extreme conditions of rainfall deficits. The proposed method is applied to six rainfall gauge stations in Iran to assess single-site drought properties as well as the inter-connection of droughts among stations.

The paper is organized as follows: The study area of Iran and data sources are described in Section 2. The methodology, including the definition of drought in terms of the SPI, the bivariate model constructed by empirical copulas, and the tail dependence coefficient used to evaluate spatial relationship of droughts, are presented in Section 3. Section 4 describes the results. Finally, summary and conclusions are given in Section 5.

2 Study Area and Data Sources

This study is carried out in Iran, situated approximately between 45° and 65°E in longitude and 25°–40°N in latitude. Iran consists of rugged, mountainous rims surrounding high interior basins. One of the major mountain ranges is the Zagros Mountains, which bisect Iran from northwest to southeast. Another high Alborz Mountains rim the Caspian Sea. The highest peak of Iran, with an elevation of 5,628 m, is located in the center of the Alborz Mountains. These high mountains also impede Mediterranean moisture crossing through the country. Iran generally has an arid climate with an average annual rainfall of 25 cm, mostly distributed from November to May. However, major exceptions are the areas located in the higher mountain valleys of the Zagros and the Caspian coastal plain, where mean annual rainfall exceeds 50 cm.

Modarres and Silva [20] have indicated that great annual rainfall variability exists in Iran. Iran can be subdivided into eight homogeneous regions, demonstrated in Fig. 1, according to the cluster analysis associated with L-moments for rainfall data [19]. A total of six rainfall gauge stations with at least 48-year monthly records are selected in this study to investigate multi-site drought relationship in Iran. The locations of these six rainfall gauge stations are also shown in Fig. 1. The monthly rainfall distribution for these six rainfall gauge stations are illustrated in Fig. 2. Clearly uneven temporal distribution as well as spatial variability are observed.

Some basic statistics, including mean, standard deviation, and coefficient of variation of annual rainfall records, for these rainfall gauge stations are reported in Table 1. The mean annual rainfall of the Rasht station, located in the Caspian coastal plain, is 1,353 mm. It is the greatest mean value among the six stations. The coefficient of variation of the Rasht station is 0.21, the lowest among the six stations. On the contrast, the Zahedan station located in southeastern Iran has the smallest mean annual rainfall of 95 mm but associated with the highest coefficient of variation of 0.42. The results show that less annual variation exists in the Rasht station, although it has greater mean annual rainfall. The Zahedan station receives scant rainfall every year but has greater fluctuation of annual rainfall. The region G1, where Isfahan and Zahedan are located, is categorized as the arid and semiarid region by Modarres [19]. These two stations receive the least annual mean rainfall among the six stations.

3 Methodology

3.1 Calculation of the SPI

To investigate the drought characteristics at different climate regions, the rainfall data need to be transformed to the SPI first. The SPI, initiated by McKee et al. [18], is developed to define and monitor droughts using only rainfall data. The

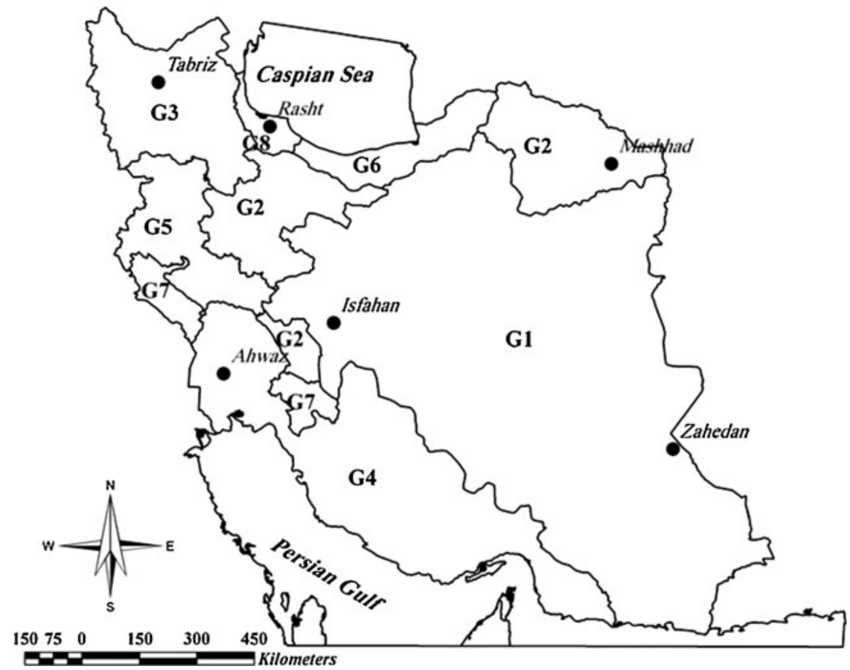


Fig. 1 Geographical illustration of the homogeneous rainfall regions and the selected rainfall gauge stations in Iran

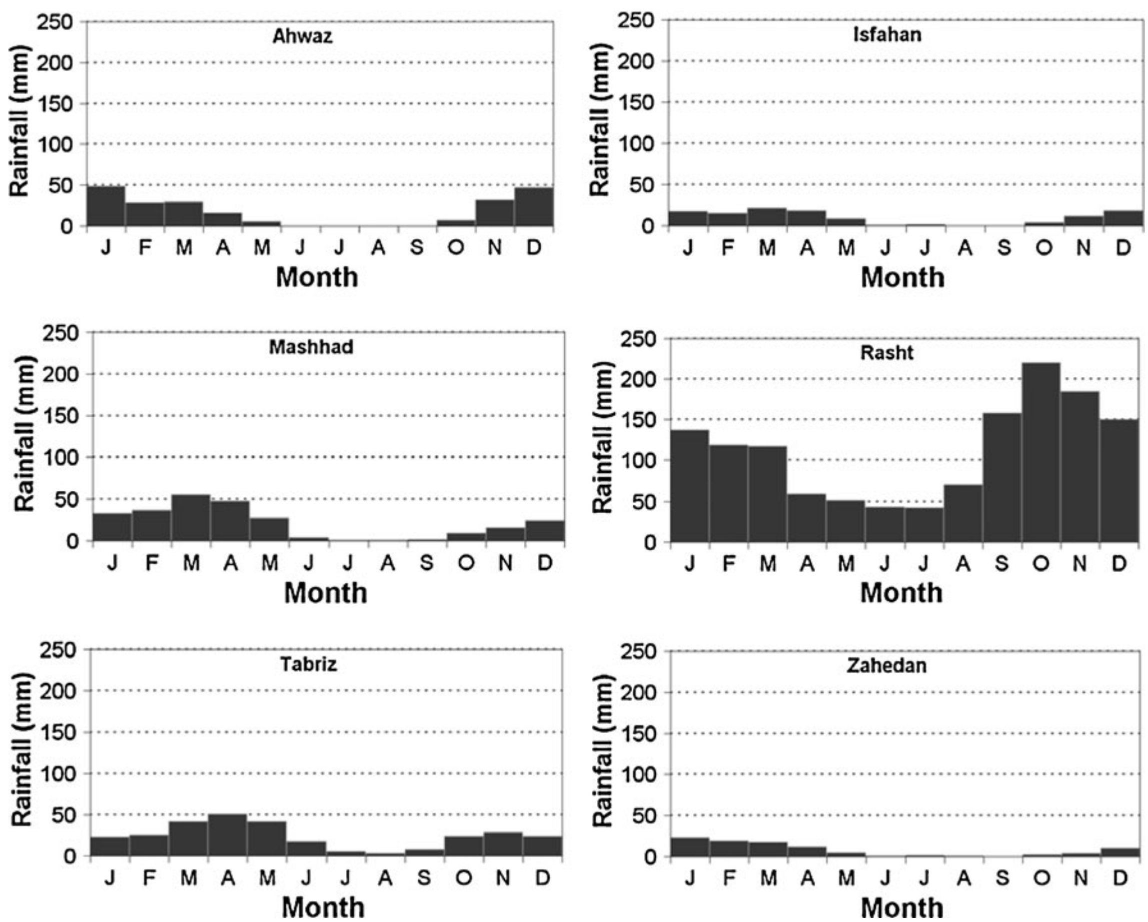


Fig. 2 Mean monthly rainfall distribution for the selected stations in Iran

Table 1 The geographical location and annual rainfall characteristics for the selected rainfall gauge stations

Station	Location		Elevation (m)	Annual rainfall		
	Latitude	Longitude		Mean (mm)	Standard deviation (mm)	Coefficient of variation
Ahwaz	31°20′	48°40′	22	213.3	86.3	0.40
Isfahan	32°37′	51°40′	1,550	121.4	40.1	0.33
Mashhad	36°16′	59°38′	999	257.5	77.4	0.30
Rasht	37°12′	49°39′	36	1,353.0	279.3	0.21
Tabriz	38°05′	46°17′	1,361	293.3	68.0	0.23
Zahedan	29°28′	48°40′	1,370	94.8	40.1	0.42

calculation of the SPI is based on monthly precipitation series aggregated at different time scales. Fitting this different time-scale monthly precipitation records to a probability distribution is the first step. Once the probability distribution is determined, the cumulative probability of observed rainfall is computed and then inverse-transformed into a standard normal distribution with mean 0 and variance 1. The resulting quantile is the SPI, which is expressed as

$$\mathrm{SPI}_i = \Phi^{-1}(F_R(r_i)) \tag{1}$$

where r_i is the aggregated rainfall data observed at month i, F_R is the cumulative distribution function (cdf) of rainfall data, Φ is the cdf of the standard normal distribution, and Φ^{-1} is the inverse of the standard normal cdf, that is, $\Phi(z)=p$ and $\Phi^{-1}(p)=z$.

A detailed description of the calculation procedure can be found in Guttman [10]. The SPI is used to quantify the rainfall deficit in terms of the probability for multiple time scales. Positive SPI indicates that the observed rainfall is greater than the median, while negative SPI means that it is below the median. The drought categories for different negative SPI values suggested by McKee et al. [18] are shown in Table 2.

3.2 Definition of Drought Severity and Duration

McKee et al. [18] defined a drought event as a period in which the SPI is continuously negative and reaches a value of −1.0 or less, as depicted in Fig. 3. So, a drought event begins when the SPI first falls below 0 and ends with positive SPI values. Two important characteristics, that is, drought severity and drought duration, are abstracted from drought events. Drought duration denoted by D, hereafter, is the continuously negative SPI period, while drought severity denoted by S is the cumulative values of the SPI within the drought duration. For convenience, drought severity is taken to be positive and defined by

$$S = -\sum_{i=1}^{D} \mathrm{SPI}_i \tag{2}$$

The definition of droughts adopted in this study excludes minor droughts, that is, continuously negative SPI periods without any SPI less than −1.0. In this study, drought events are characterized by drought severity and duration simultaneously; a bivariate distribution is therefore required to model the observed drought data.

Table 2 Drought categories defined by the SPI values

SPI values	Drought category
0 to −0.99	Near normal or mild drought
−1.00 to −1.49	Moderate drought
−1.50 to −1.99	Severe drought
≤−2.00	Extreme drought

3.3 A Bivariate Drought Model Constructed by Empirical Copulas

Correlated drought severity and drought duration may not be fitted by the same type of distribution. Commonly used bivariate distributions such as the bivariate extreme value model cannot be used. Construction of a bivariate distribution using copulas, developed by Sklar [38], can overcome such difficulties arising in practical applications. The essential concept of copulas is that a joint distribution of correlated random variables can be expressed as a function of the univariate marginal distributions for each random variable. That is, copulas are functions that are able to model the dependence structure among correlated random variables regardless of the marginal distributions.

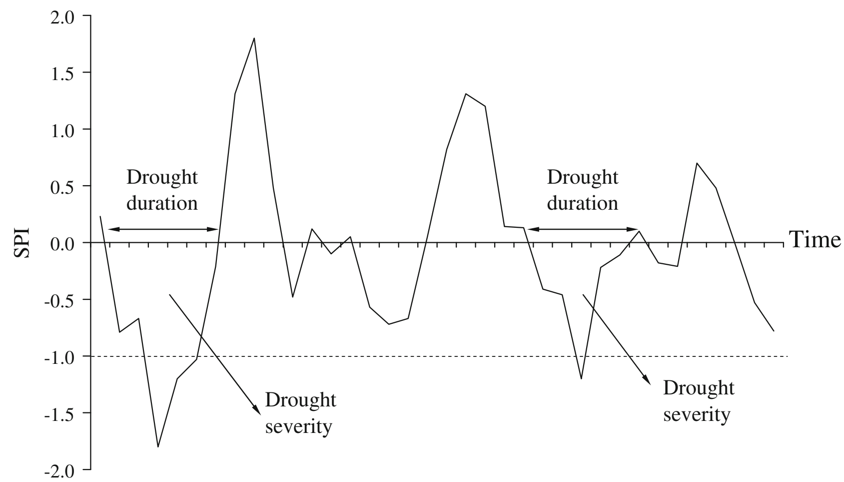
In terms of correlated drought severity S and drought duration D, Sklar's theorem states that if $F_{S,D}$ (s,d) is a bivariate cdf of S and D with respective marginal cdfs F_S (s) and F_D (d), then there exists a copula C such that

$$F_{S,D}(s,d) = C(F_S(s),F_D(d)) \tag{3}$$

Conversely, for univariate marginal cdfs F_S (s) and F_D (d) and any copula C, the function $F_{S,D}$ (s,d) is a joint cdf with marginal cdfs F_S (s) and F_D (d).

Detailed properties and various types of copulas can be founded in Joe [12], Nelsen [21], Cherubini et al. [3], and

Fig. 3 Schematic definition of drought events by the SPI series



Salvadori et al. [29]. Although copulas offer great flexibility to construct multivariate models, incorrect choices of the marginal distributions and copulas could lead to unreliable outcomes. In this study, an empirical formulation is adopted to model the drought severity and duration and the dependence structure between them. The empirical copula is a rank-based measure of multivariate distribution functions. In terms of drought severity and duration, the empirical copula can be expressed as [21]

$$\widehat{F}_{S,D}(s,d) = \widehat{C}\left(\widehat{F}_S(s),\widehat{F}_D(d)\right) \tag{4}$$

where $\widehat{C}$ denotes the empirical copula; $\widehat{F}_{S,D}(s,d)$ is the empirical distribution function of drought severity and duration given by

$$\widehat{F}_{S,D}(s,d) = \widehat{C}\left(\widehat{F}_S(s),\widehat{F}_D(d)\right) = \frac{1}{n}\sum_{i=1}^{n} I\left(s \le s_{(i)}, d \le d_{(i)}\right) \tag{5}$$

where n is the sample size; $I\ (\cdot)$ denotes the indicator function; $s_{(i)}$ and $d_{(i)}$, $1 \le i \le n$ denote order statistics of the observed drought severity and duration.

$\widehat{F}_S(s)$ and $\widehat{F}_D(d)$ in Eq. 5 are the empirical univariate distribution functions of drought severity and duration:

$$\widehat{F}_S(s) = \frac{1}{n}\sum_{i=1}^{n} I\left(s \le s_{(i)}\right) \tag{6a}$$

$$\widehat{F}_D(d) = \frac{1}{n}\sum_{i=1}^{n} I\left(d \le d_{(i)}\right) \tag{6b}$$

3.4 Multi-sites Drought Relationship by Tail Dependence

Simultaneous occurrences of severe droughts at different sites can deteriorate water deficit problem. Since the dependence between droughts (say values of the SPI less than −1.0) at different sites may be significantly different than that of non-droughts (SPI values≥−1.0), the tail dependence is employed to explore the multi-sites drought connections. The tail dependence measures the degree of association in the upper-right quadrant and the lower-right quadrant of a bivariate distribution [12]. Serinaldi [31] has studied the dependence for ten different time scales rainfall data (0.5 to 24 h) using the upper tail dependence coefficient. Only the lower tail dependence is considered in this study since drought events (negative SPIs with one SPI less than −1.0) are evaluated.

For a bivariate distribution function $F_{X,Y}\ (x,y)$, the lower tail dependence coefficient is defined as

$$\lambda_L = \lim_{w \to 0^+} P(F_X(x) \le w | F_Y(y) \le w) \tag{7}$$

where $F_X\ (x)$ and $F_Y\ (y)$ are the marginal distribution functions of X and Y, respectively. Schmidt and Stadtmuller [30] suggest the following nonparametric estimate of the lower tail dependence coefficient:

$$\lambda_L = \frac{1}{k}\sum_{i=1}^{n} I\left(R_X^i \le k, R_Y^i \le k\right) \tag{8}$$

where k is the threshold rank and R_X^i and R_Y^i denote the ranks of the ith observations of X and Y, respectively.

The variables X and Y are said to be lower tail independent if $\lambda_L=0$ and lower tail dependent if $0 < \lambda_L \le 1$. In this study, the lower tail dependence coefficients for different time-scale SPI series are pairwisely calculated to explore the spatial connection of droughts.

4 Results and Discussions

4.1 Basic Statistics of Observed Droughts for Selected Stations

The monthly rainfall records for the six selected rainfall gauge stations in Iran, listed in Table 1, are reformatted as five different time-scale rainfall series, including 3-, 6-, 9-, 12-, and 24-month rainfall series. These five different time-scale rainfall series are fitted by gamma distributions for each rainfall gauge station and then transformed to different time-scale SPI values by Eq. 1. Figure 4 shows the different time-scale SPI series of the period 2000–2003 for each station. Generally, the shorter time-scale SPI series has greater fluctuation, caused by larger number of drought events with shorter drought durations. On the other hand, the longer time-scale SPI series exhibits smooth variation, resulting in longer drought durations.

Drought events for each station and each time-scale SPI series are then abstracted using the definition mentioned previously, that is, continuously negative SPI periods with the SPI reaching −1.0 or less. Some basic drought statistics, including mean, standard deviation, maximum, and minimum of drought severity and duration, for each station and each time scale are reported in Table 3. The arrival rates of droughts for these six stations are also listed in Table 3. The least frequent occurrence of droughts is observed at the Ahwaz station for all time scales. The mean values of drought duration for the Ahwaz station are generally greater than those of other stations. For example, the mean drought duration for the 6-, 9-, and 24-month SPI series are the greatest for the Ahwaz station. This result demonstrates that whenever a drought occurs at the Ahwaz station, it would generally sustain for a longer period. On the other hand, the stations associated with shorter mean drought durations generally have more frequent occurrences of droughts. For instance, the Rasht station has the smallest mean drought durations, but its drought arrival rate is ranked first among the six stations for all time scales. This fact indicates that the climate would quickly return to a normal state whenever droughts occur at the Rasht station.

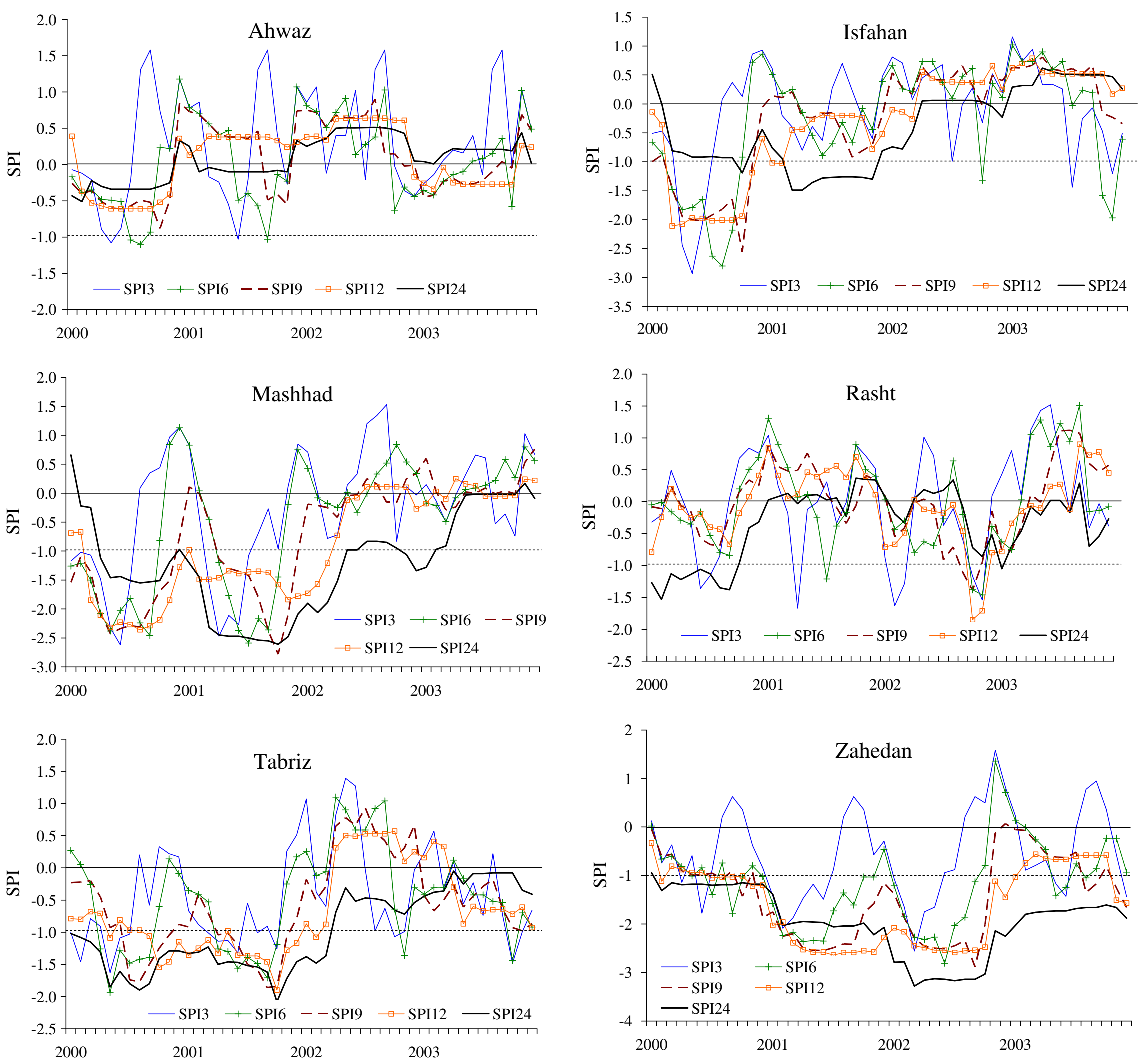


Fig. 4 Different time-scale SPI series of the period 2000–2003 for the selected stations

Table 3 Basic statistics of drought data for the selected stations in Iran

Station			Ahwaz	Isfahan	Mashhad	Rasht	Tabriz	Zahedan
No. of years			53	53	53	48	53	53
3 months	No. of droughts		23	35	35	41	35	34
	Arrival rate		0.434	0.660	0.660	0.854	0.660	0.642
	Drought severity	Mean	4.67	4.88	5.59	5.07	6.08	4.10
		SD	3.15	2.81	3.44	3.14	4.53	2.40
		Max	10.60	11.72	13.73	15.17	21.16	11.62
		Min	1.23	1.62	1.23	1.54	1.03	1.34
	Drought duration	Mean	5.74	6.00	6.09	4.95	6.34	4.74
		SD	2.73	3.33	3.84	2.92	3.91	2.23
		Max	10.00	13.00	15.00	12.00	18.00	9.00
		Min	2.00	1.00	1.00	1.00	1.00	2.00
6 months	No. of droughts		15	23	28	24	20	28
	Arrival rate		0.283	0.434	0.528	0.500	0.377	0.528
	Drought severity	Mean	10.41	9.69	8.23	8.00	10.49	7.96
		SD	18.54	7.17	6.40	4.95	5.87	9.30
		Max	76.55	31.04	29.45	20.71	24.54	49.48
		Min	2.29	1.10	1.21	1.10	2.78	1.39
	Drought duration	Mean	11.73	10.00	8.37	7.79	10.90	9.50
		SD	14.24	6.21	5.17	3.86	4.68	7.36
		Max	62.00	28.00	22.00	18.00	19.00	33.00
		Min	3.00	1.00	1.00	1.00	4.00	2.00
9 months	No. of droughts		11	15	20	19	16	20
	Arrival rate		0.208	0.283	0.377	0.396	0.302	0.377
	Drought severity	Mean	14.94	15.78	11.99	10.60	13.40	11.90
		SD	25.74	14.49	8.81	6.55	13.54	15.59
		Max	91.15	54.86	34.55	25.73	55.36	72.93
		Min	2.94	2.84	2.98	3.94	1.25	1.97
	Drought duration	Mean	16.82	16.67	12.45	10.58	14.13	14.30
		SD	19.51	11.21	6.98	5.00	14.19	11.55
		Max	70.00	45.00	35.00	24.00	63.00	52.00
		Min	3.00	4.00	4.00	4.00	1.00	2.00
12 months	No. of droughts		7	8	12	15	8	9
	Arrival rate		0.132	0.151	0.226	0.313	0.151	0.170
	Drought severity	Mean	22.54	29.22	20.15	13.93	25.70	23.99
		SD	31.25	23.91	14.11	11.19	27.77	28.19
		Max	90.75	80.82	45.90	34.48	83.41	87.84
		Min	1.78	7.51	2.37	1.51	5.00	1.99
	Drought duration	Mean	27.14	29.50	22.17	15.6	25.38	27.33
		SD	23.28	21.40	13.45	10.20	25.50	22.14
		Max	71.00	75.00	44.00	33.00	83.00	61.00
		Min	3.00	12.00	4.00	2.00	9.00	4.00
24 months	No. of droughts		3	4	6	7	4	4
	Arrival rate		0.057	0.075	0.113	0.146	0.075	0.075
	Drought severity	Mean	48.56	54.80	39.89	28.83	54.95	50.98
		SD	39.57	63.15	16.26	23.78	35.85	33.14
		Max	92.90	149.39	58.72	67.72	102.04	95.23
		Min	16.84	18.61	16.04	1.38	35.85	15.12
	Drought duration	Mean	59.00	52.00	42.00	29.29	58.75	45.25
		SD	13.00	39.98	22.29	19.27	29.57	13.89
		Max	72.00	110.00	82.00	56.00	97.00	56.00
		Min	46.00	25.00	15.00	3.00	29.57	25.00

4.2 Copula-Based Bivariate Drought Models

The recurrence intervals of drought severity and duration (separately) are related to the univariate empirical distributions of drought severity and duration, respectively, by

$$T_S = \frac{1}{\gamma(P(S \geq s))} = \frac{1}{\gamma\left(1 - \widehat{F}_S(s)\right)} = \frac{1}{\gamma\left(1 - \frac{1}{n}\sum_{i=1}^{n} I\left(s \leq s_{(i)}\right)\right)} \tag{9a}$$

$$T_D = \frac{1}{\gamma(P(D \geq d))} = \frac{1}{\gamma\left(1 - \widehat{F}_D(d)\right)} = \frac{1}{\gamma\left(1 - \frac{1}{n}\sum_{i=1}^{n} I\left(d \leq d_{(i)}\right)\right)} \tag{9b}$$

where T_s and T_d denote the recurrence intervals (in years) of drought severity and duration, respectively; γ is the arrival rate of drought events, that is, the number of drought events per year.

The corresponding 5-, 10-, 25-, and 50-year recurrence intervals for 3-, 6-, and 9-month SPI series for each station are summarized in Table 4. The recurrence intervals for 12- and 24-month SPI series are not calculated because of few samples. No consistent trends are observed among the various recurrence intervals for the six stations. Generally, the least drought severity and duration for the same recurrence intervals are observed at the Ahwaz station since it has the lowest arrival rate of droughts. The greatest drought severity and duration for the same recurrence intervals are observed at the other five stations. For shorter recurrence intervals, the greatest drought severity and duration are observed at the Isfahan, Mashhad, and Tabriz stations. The greatest drought severity and duration for longer recurrence intervals are observed at the Zahedan station.

In this study, the empirical copula defined in Eq. 5 is used to construct the bivariate distribution of drought severity and duration. The contours of probabilities for the six stations for the 3-month SPI series are shown in Fig. 5. Non-smooth contours are caused by limited samples and linear interpolation. The empirical copulas of drought severity and duration for 6-, 9-, and 12-month SPI series are not shown because they appear similar. It is difficult to construct contours for the 24-month SPI series because of limited samples.

Bivariate frequency of drought severity and duration in terms of recurrence intervals can be derived as [37]:

$$T_{SD} = \frac{1}{\gamma P(S > s \text{or} D > d)} = \frac{1}{\gamma\left(1 - F_{S,D}(s,d)\right)} = \frac{1}{\gamma\left(1 - \frac{1}{n}\sum_{i=1}^{n} I\left(s \leq s_{(i)}, d \leq d_{(i)}\right)\right)} \tag{10}$$

Table 4 Drought severity and duration for various univariate recurrence intervals determined separately

Station			Ahwaz	Isfahan	Mashhad	Rasht	Tabriz	Zahedan
3 months	Drought severity	5 years	3.59	5.33	6.84	7.39	7.30	4.73
		10 years	5.46	7.79	8.56	9.26	10.43	5.48
		25 years	10.52	10.52	12.29	10.07	13.54	6.77
		50 years	10.60	11.07	12.71	12.07	14.04	11.08
	Drought duration	5 years	5.00	6.79	7.39	6.00	7.39	6.00
		10 years	8.40	9.70	9.70	8.60	10.00	7.00
		25 years	10.00	12.00	12.88	11.08	11.88	8.88
		50 years	10.00	12.94	13.94	12.00	12.94	9.00
6 months	Drought severity	5 years	4.27	7.51	8.52	8.72	8.42	7.06
		10 years	5.62	13.48	11.22	12.28	14.56	9.10
		25 years	7.61	19.11	18.85	14.16	16.67	12.83
		50 years	14.46	21.44	20.30	16.03	17.22	21.77
	Drought duration	5 years	6.00	9.40	9.00	8.40	9.39	9.39
		10 years	8.70	13.40	12.00	11.00	12.69	11.00
		25 years	12.88	15.88	13.88	11.08	17.88	19.76
		50 years	13.94	18.82	19.64	12.24	18.00	26.58
9 months	Drought severity	5 years	2.88	6.70	8.61	7.69	4.71	9.85
		10 years	5.95	14.53	12.25	11.72	15.70	10.37
		25 years	13.79	20.25	22.56	19.24	19.63	24.30
		50 years	17.91	40.97	32.34	24.58	27.55	37.43
	Drought duration	5 years	1.85	10.40	11.00	10.40	8.00	11.42
		10 years	10.00	14.70	12.69	12.00	14.81	17.42
		25 years	12.00	22.52	18.76	13.57	18.76	29.91
		50 years	33.67	37.10	21.82	20.16	19.94	37.61

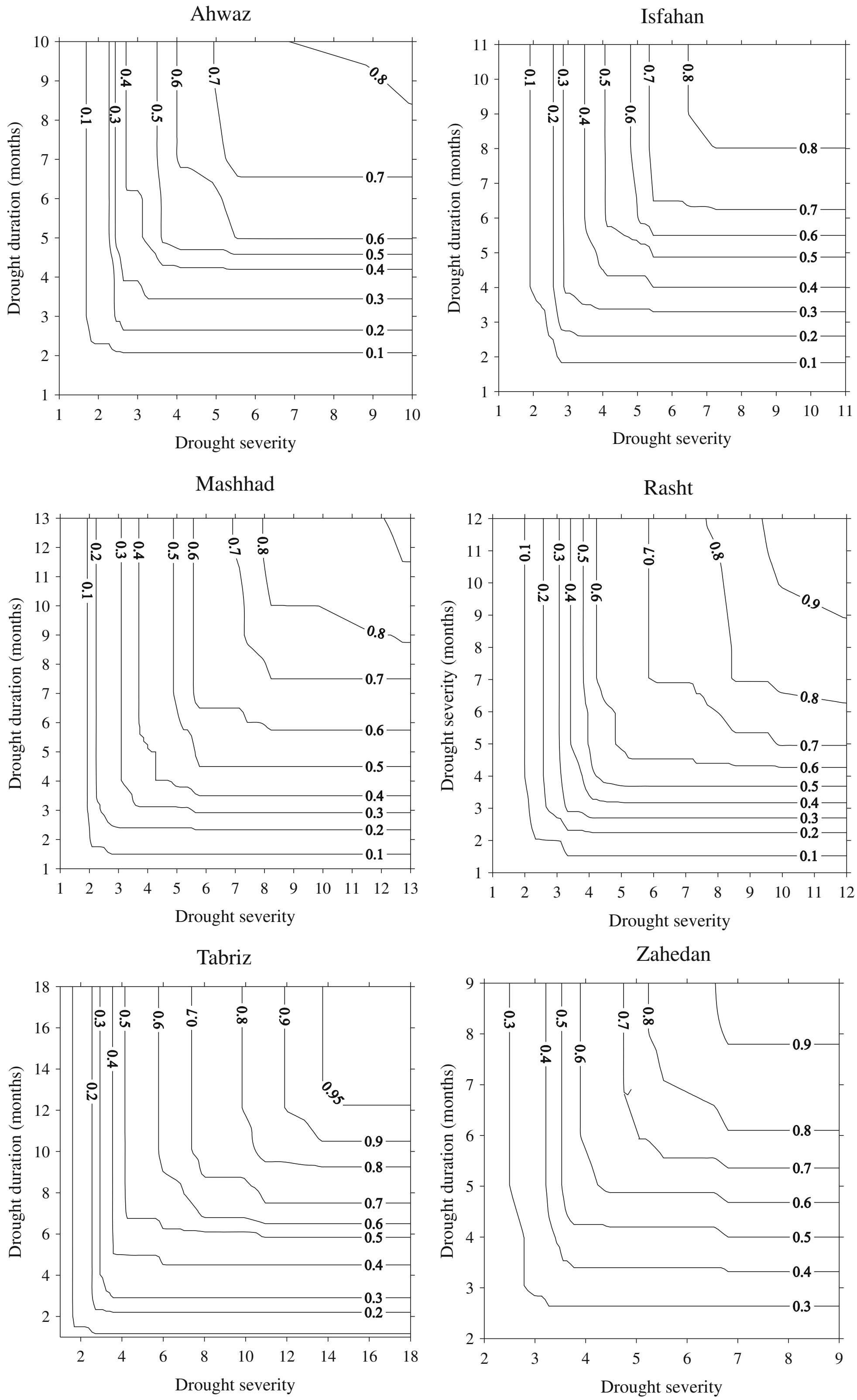


Fig. 5 Empirical copulas for the selected stations for the 3-month SPI series

where T_{SD} denotes the recurrence interval of drought severity exceeding a specific value and drought duration exceeding another specific value and γ is the arrival rate of drought events. The values of γ for different time-scale SPI series and for each station are reported in Table 3.

The contours for various recurrence intervals for the 3-month SPI series are shown in Fig. 6. The contours for other

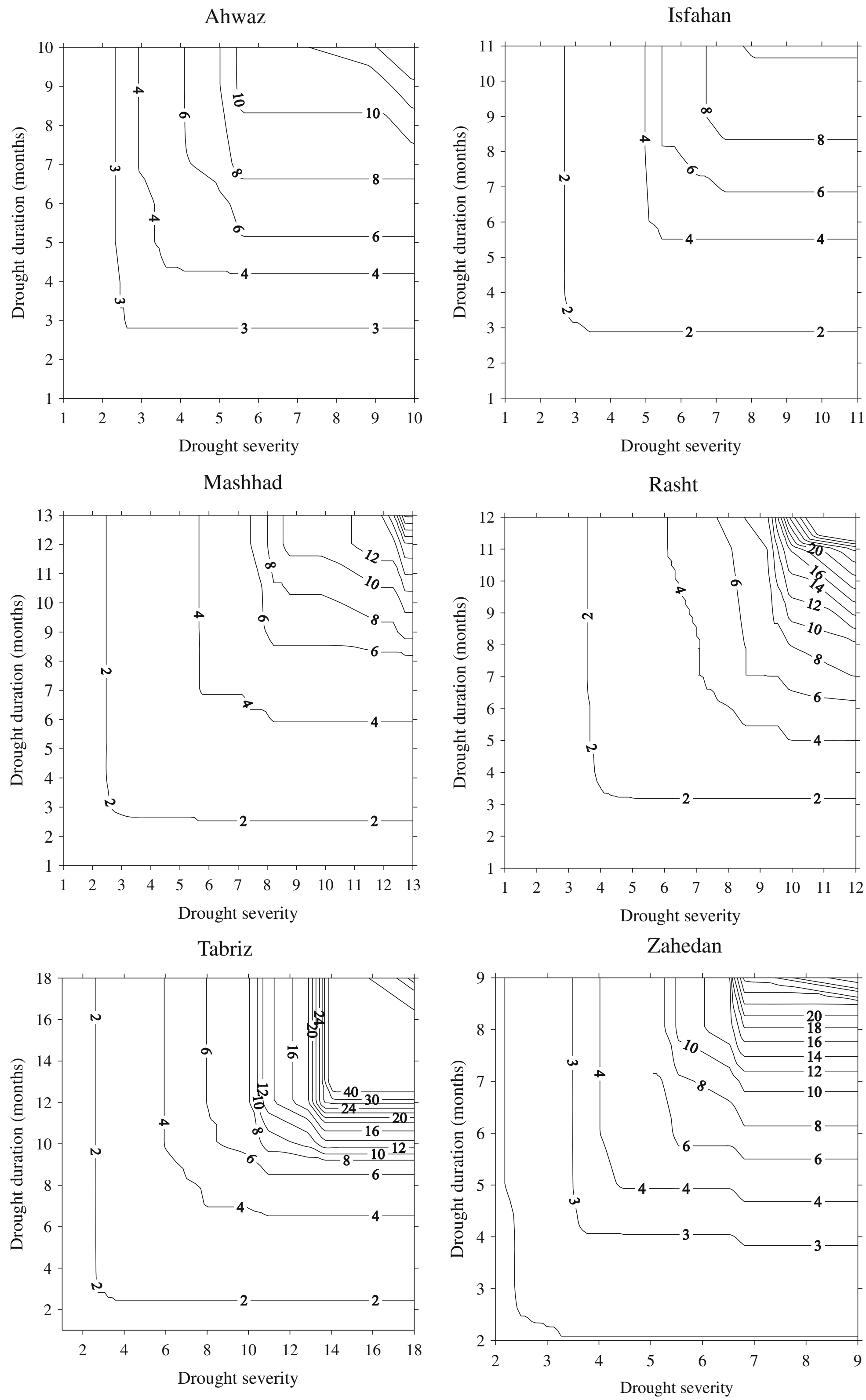


Fig. 6 Contours of recurrence intervals (years) for the selected stations for the 3-month SPI series

time-scale SPI series are not shown because they appear similar. Figure 6 can be used to compare the recurrence intervals of a specific drought event across different stations. For example, the recurrence interval of a drought event with a severity of 5 and a duration of 5 months is approximately equal to 5 years at the Ahwaz station, 4 years at the Isfahan and Zahedan stations, and less than 3 years at the other three stations.

4.3 Tail Dependence Coefficient Analysis

The lower tail dependence coefficients (TDC) are estimated pairwisely for the six stations for various time-scale SPI series using the nonparametric method defined in Eq. 8. Percentiles of 5, 10, 15, 20, and 25 are considered in this study to estimate the TDC. Figure 7 demonstrates the effects of percentiles on the TDCs for each station. Generally, the TDCs for the 3-month SPI series are greater. The TDCs generally reduce for longer durations of the SPI series. However, some TDCs increase for longer durations of the SPI series. The figure also shows that the TDCs increase with increasing percentiles.

Figure 8 demonstrates the relationship of the TDCs with distances. The TDCs slightly decrease with distances for the 3-month SPI series. However, the TDCs for other time-scale SPI series do not exhibit specific trends. Generally, distance is not a key parameter for the multi-sites drought connections in Iran. This fact implies that droughts may occur simultaneously at two distant regions. On the other hand,

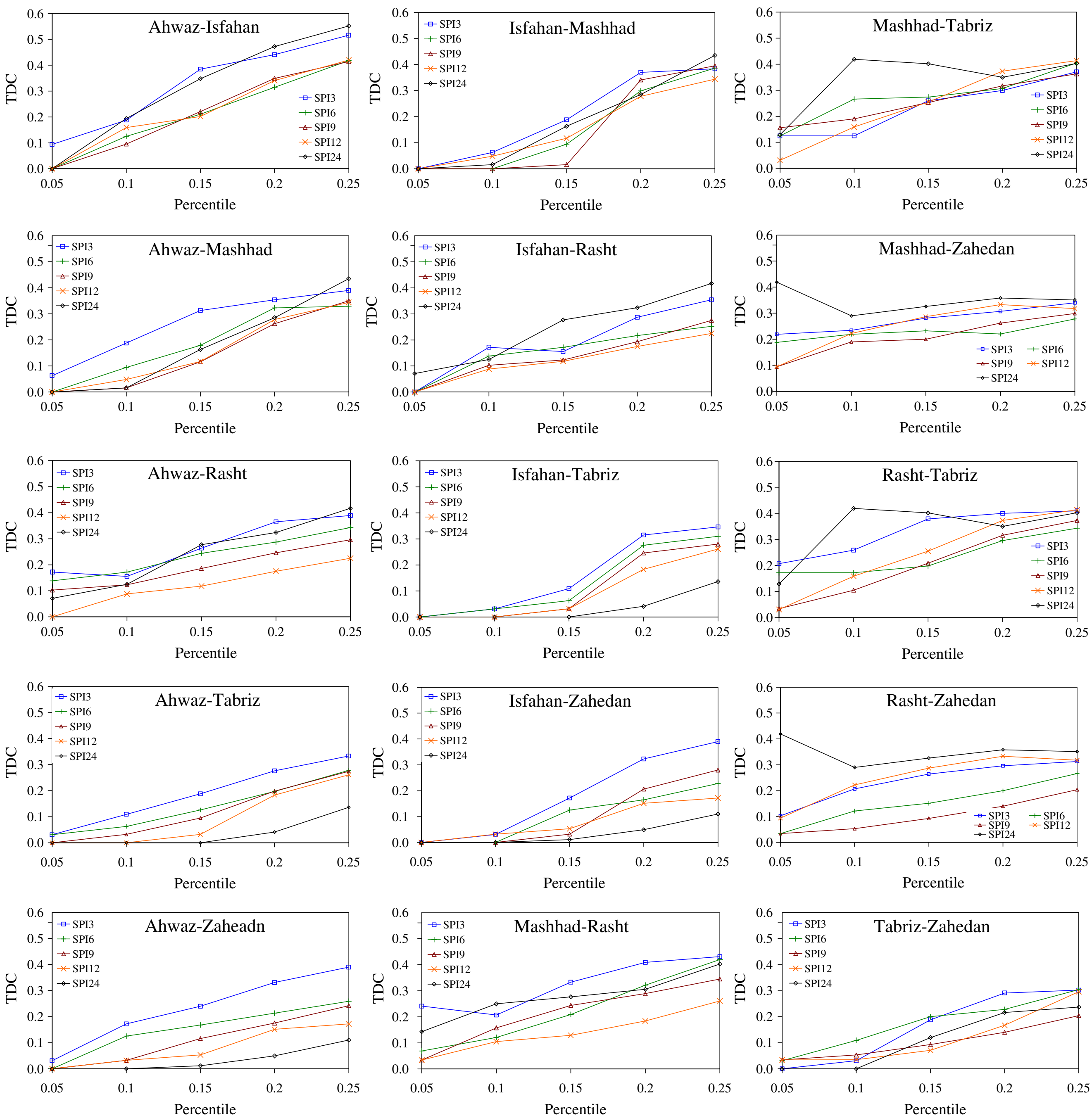


Fig. 7 Pairwise tail dependence coefficient for the selected stations for different time-scale SPI series

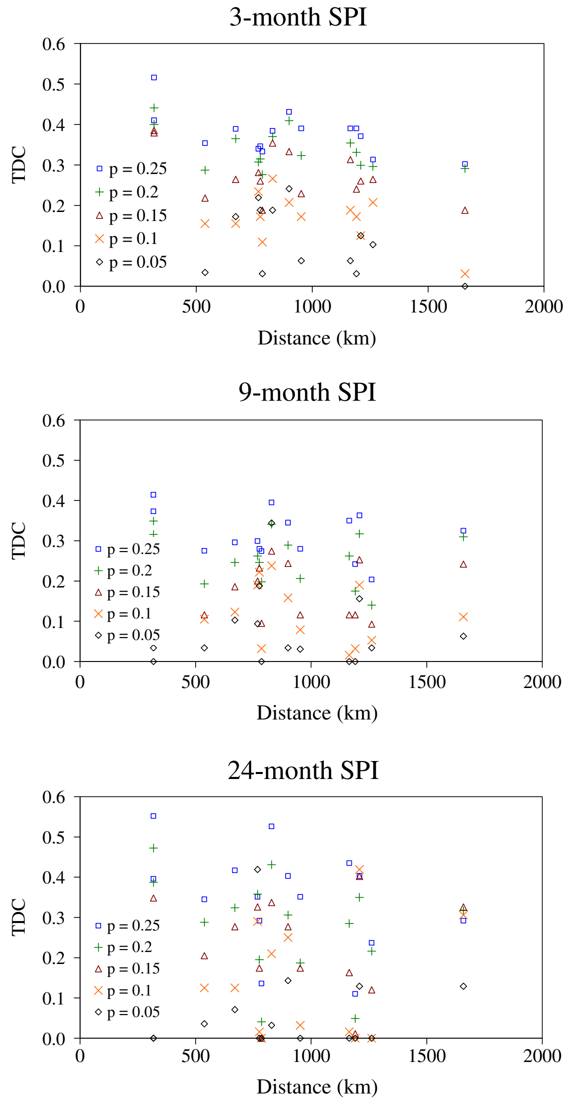


Fig. 8 The tail dependence coefficient versus distance for various percentiles for different time-scale SPI series

nearby regions may not have simultaneous droughts. According to Figs. 7 and 8, the Ahwaz and Isfahan stations have the highest probability to have droughts simultaneously. Although the distance between the Rasht and Tabriz stations is approximately equal to the distance between the Ahwaz and Isfahan stations, the TDCs between Rasht and Tabriz are lower. The lowest TDCs between Tabriz and Zahedan observed in Fig. 7 show that they have the lowest probability to have droughts simultaneously.

5 Summary and Conclusions

This study uses an empirical copula function to construct a bivariate drought severity and duration distribution for six stations in Iran. Droughts are defined as continuously negative SPI periods with one SPI reaching −1 or less. Various time-scale SPI series, including 3, 6, 9, 12, and 24 months, are employed in this study. Bivariate drought frequency analyses performed by the established empirical copula show that the Ahwaz station has the lowest values of drought severity and duration for various recurrence intervals. For shorter duration SPI series, the Mashhad, Rasht, and Tabriz stations have greater values of drought severity and duration. For longer-duration SPI series, the Isfahan and Zahedan stations have greater values.

Inter-connection of droughts among stations is explored by the lower tail dependence coefficient. Estimated by a nonparametric method, various percentiles of the TDCs are established pairwise among the six stations. The results show that the TDCs reduce with percentiles. The distance between stations is not a key parameter for inter-connection of droughts in Iran. Generally, the Ahwaz and Isfahan stations have the highest probability to have droughts simultaneously. On the other hand, the Tabriz and Zahedan stations have the lowest probability to have droughts simultaneously.

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