Water Resour Manage (2010) 24:2365–2380
DOI 10.1007/s11269-009-9556-5

Regional Dry Spells Frequency Analysis by L-Moment and Multivariate Analysis

Reza Modarres

Received: 22 April 2009 / Accepted: 6 December 2009 /
Published online: 15 December 2009

Abstract The spatial variation of the statistical characteristics of the extreme dry events, such as the annual maximum dry spell length (AMDSL), is a key practice for regional drought analysis and mitigation management. For arid and semi arid regions, where the data set is short and insufficient, the regionalization methods are applied to transfer at-site data to a region. The spatial variation and regional frequency distribution of annual maximum dry spell length for Isfahan Province, located in the semi arid region of Iran, was investigated using a daily database compiled from 31 rain gauges and both L-moment and multivariate analysis. The use of L-moment method showed a homogeneous region over entire province with generalized logistic distribution (GLOG) as the regional frequency distribution. However, the cluster analysis performed two regions in west and east of the province where L-moment method demonstrated the homogeneity of the regions and GLOG and Pearson Type III (PIII) distributions as regional frequency distributions for each region, respectively. The principal component analysis was applied on at-site statistics of AMDSL and found the L-coefficient of skewness (LCs) and maximum AMDSL the main variables describing the spatial variation of AMDSL over the Isfahan Province. The comparison of two homogeneous regions also proved the difference between two regions. Therefore, this study indicates the advantage of the use of multivariate methods with L-moment method for hydrologic regionalization and spatial variation of drought statistical characteristics.

Keywords Drought · Dry spells · Regionalization · L-moments · L-Coefficient of skewness · GLOG distribution · Multivariate methods · Nonparametric tests

R. Modarres (✉)
INRS-ETE, 490 de la Couronne, Quebec, Qc, G1K 9A9, Canada
e-mail: Reza.Modarres@ete.inrs.ca

1 Introduction

Drought is considered to be the most complex but least understood natural hazard with a major impact on socio-economic and agricultural activities and significant temporary reduction in water availability. Drought analysis may be made on the basis of single site data (Dracup et al. 1980) and multi-site data (Guttman et al. 1992), depending on the purpose of the study. In this paper, we are concerned with regional drought, so our analysis will be based on spatial data measured at several sites.

The identification of local and regional drought events has been studied by numerous investigators and several drought indices have been used for drought analysis. Byun and Wilhite (1999) classified the study of drought into four categories, understanding the causes of drought, understanding the frequency and severity of drought in order to characterize the probability of occurrence of drought of various magnitudes, understanding the impacts of droughts and finally, finding the responses and appropriate mitigation strategies for reduction drought impacts.

The site estimation of drought probabilities at gauged basins is not a difficult task for hydrologists. In contrary, regional drought frequency analysis, which fits distribution and estimates frequency or return period for a specific region, has been always a problem for hydrologists.

A number of regionalization methods have been developed. These techniques can be classified into two different types (Durrans and Tomic 1996), the first is devoted to the prediction in ungauged basins (PUB) in which the relationship of certain hydrological characteristics (e.g. the low flow) with physiographic and climate characteristics is established for gauged basins and then the relationship is applied to predict the hydrological characteristics for ungauged basins. These techniques include multivariate statistics such as regression, hierarchical cluster and principal component analyses (e.g. Bates et al. 1998; Chiang et al. 2002; Modarres 2006), region of influence (Burn 1990) and canonical correlation analysis (Cavadias et al. 2001).

The second type of regionalization technique is referred to as regional frequency analysis or pooled frequency analysis (Reed et al. 1999). Several methods for regional frequency analysis have been proposed, most of them try to identify homogeneous regions using the concept of regional homogeneity, choose suitable distribution and estimate accurate parameters for the model. Among different methods of regional frequency analysis, the method of L-moments has been used increasingly by hydrologists.

L-moments method has been used for the estimation of regional flood frequency distribution (e.g. Vogel and Wilson 1996; Parida et al. 1998; Kjeldsen et al. 2002; Lim and Lye 2003, and others), regional low flow frequency analysis (e.g. ARIDE 1999; Kroll and Vogel 2002; Chen et al. 2006; Modarres 2008) and rainfall frequency analysis (e.g. Wallis et al. 2007) because it is a robust and unbiased method for estimation regional parameters. However, this method has not been widely used for dry spell frequency analysis. Dry spells, or the consecutive days without rainfall or days with rainfall below a defined threshold, are considered as an extreme dry event with major impacts on agriculture due to its impact on soil moisture deficit after several days. In previous studies on dry spell frequency analysis, Lana and Burgueno (1998a) applied Gumbel distribution, the transition probability from wet to dry spells was investigated by Martin-Vide and Gomez (1999) using markov chains and repeated long dry spells by the Poisson distribution which was investigated by Lana and Burgueno (1998b). More recently, Vicente-Serrano and Begueria-Portugues

(2003) applied Gumbel and Pareto distribution to estimate the annual maximum dry spell length in different return periods in order to estimate the risk of longest dry spell period in Spain. Lana et al. (2006) fitted Generalized Extreme Value distribution to AMDSL of Norteast Spain and generalized Pareto distribution to dry spell length of different daily rainfall thresholds of 0.1, 1.0, and 5.0 mm/day. In both cases, the estimation of the parameters was obtained by L-moments.

While the L-moment method is increasingly being used for regional frequency analysis, on one hand, this method has not been a popular tool among Iranian researchers. On the other hand, although Iran is located in the arid and semi arid belt of the world and drought is a common feature in the country but no study on dry spells has been reported in the country.

The present work will contribute to fill such a knowledge gap in Iran applying both L-moments and multivariate methods to find homogenous regions of annual maximum dry spell length (AMDSL) of Isfahan Province in the center of Iran. The contents of the paper are structured as follows. Section 2 deals with the geographical locations and a brief description of the dataset. The pooled frequency distribution analysis of dry spells is discussed in Sections 3 and 4 using L-moments. The use of multivariate methods for dry spell regionalization is described in Section 5. In Section 6, three non-parametric methods are applied for statistical comparisons of the homogeneous groups derived from L-moments and multivariate methods. A brief conclusion is presented in the last section.

2 Data Set

Isfahan Province is located in the center of Iran (Fig. 1). The eastern regions of the province are located in the western margins of arid and semi arid regions of Iran. The western regions of the province lay in the eastern hill slopes of Zagrous

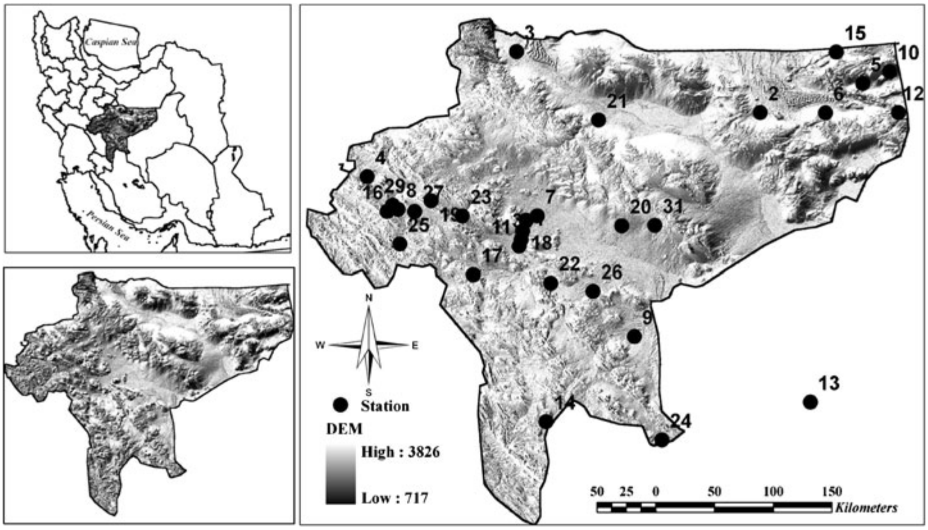


Fig. 1 Location map of the stations in Isfahan Province

Mountains. There are 31 rain gauges in Isfahan province with length of daily rainfall records, at least 14 years, which are used in this study to investigate the regional frequency distribution of dry spell in Isfahan Province. The data set of this study includes annual maximum dry spell length (AMDSL). The annual maximum dry spell length increases from 206 in the western region to 322 days in the eastern region of the province. In the regional scale, no significant trend has been found in this region (Modarres and da Silva 2007; Modarres and Sarhadi 2009). Therefore, the assumptions of randomness of dry spell could not be rejected and the frequency distribution functions could be fitted to dry spell data.

3 Pooled Frequency Analysis

3.1 Definition of L-moments

L-moments, as defined by Hosking and Wallis (1997) are linear combinations of probability weighted moments (PWM) and are defined as

$$\beta_r = E\left\{X\left[F(X)\right]^r\right\} \tag{1}$$

where β_r is the rth order PWM and F(x) is the cumulative distribution function (cdf) of the random variable X. Unbiased estimator (b_i) of the PWM are given by Hosking and Wallis (1997) as:

$$b_r = n^{-1} \sum_{j=r+1}^{n} \frac{(j-1)(j-1)\dots(j-r)}{(n-1)(n-2)\dots(n-r)} x_{j:n} \tag{2}$$

where n is the sample size and $x_{j:n}$ represents an ordered sample $x_{1:n} \leq x_{2:n} \leq x_{n:n}$ from distribution of X. The sample L-moments (l_r) are linear combinations of sample PWM calculated as:

$$l_1 = b_0 \tag{3}$$

$$l_1 = 2b_1 - b_0 \tag{4}$$

$$l_1 = 6b_2 - 6b_1 + b_0 \tag{5}$$

The sample L-moment ratios, t_r, are based on the sample L moments and defined as

$$t_2 = \frac{l_2}{l_1} \qquad t_3 = \frac{l_3}{l_2} \tag{6}$$

where t_2 is the sample L-coefficient of variation(L-Cv) and t_3 the sample L-coefficient of skewness (L-Cs) (Hosking and Wallis 1997).

3.2 Moment Ratio Diagram

Moment Ratio Diagrams (MRDs) are useful tools for visual inspection of heterogeneity of a region (Stedinger et al. 1993) and are always preferred to produce moment ratio diagrams for goodness-of-fit tests (Vogel and Fennessey 1993; ARIDE 1999). Table 1 shows the L-moments of AMDSL in the selected stations. Figures 2 and 3 show MRDs for all stations in the province. It is clear that data points are not

Table 1 L-moments ratio and discordancy statistics for AMDSL series of Isfahan Province

Station	Sample size (year)	Mean AMDSL (day)	L-Cv	L-Cs	L-Ck	Discordancy measurement
Aminabad	23	179	0.12	0.02	0.34	2.31
Anarak	18	161	0.19	0.02	0.09	0.48
Aran	18	146	0.14	−0.17	0.18	0.67
Aznaveleh	35	123	0.18	−0.03	0.09	0.38
Chahmalek	34	176	0.15	−0.09	0.17	0.24
Choopanan	34	183	0.14	0.11	0.24	1.98
Dolatabad	25	150	0.21	−0.17	0.11	1.21
Skandari	20	152	0.17	−0.04	0.15	0.02
Sfandaran	22	186	0.12	0.03	0.21	1.54
Farokhi	17	183	0.16	−0.03	0.1	0.34
Falavarjan	32	138	0.18	−0.08	0.12	0.12
Garmeh	32	181	0.15	−0.07	0.24	0.49
Hajiabad	24	169	0.17	−0.11	0.16	0.15
Hana	14	184	0.12	−0.11	0.25	1.13
Jandagh	36	166	0.19	−0.06	0.23	1.07
Kalbali	23	159	0.24	0.11	0.28	3.46[a]
Klishad	34	166	0.14	−0.17	0.22	1.3
Khomenishahr	33	156	0.16	−0.06	0.08	0.81
Kordolia	33	128	0.16	−0.07	0.06	1.12
Kohpayeh	36	188	0.18	−0.04	0.19	0.07
Mahabad	33	175	0.18	0	0.13	0.18
Mahyar	33	157	0.16	−0.16	0.17	0.64
Karvand	31	124	0.23	0.06	0.13	1.39
Mobarakeh	30	169	0.18	−0.12	0.17	0.35
Oregan	25	112	0.22	−0.13	0.13	1.44
Pjarghooyeh	35	157	0.22	−0.01	0.06	0.98
Rozveh	25	146	0.19	−0.11	0.11	0.36
Poshtbadam	25	159	0.17	0.12	0.16	1.56
Savaran	25	133	0.21	−0.08	0.01	1.14
Tad	35	158	0.14	−0.18	0.11	1.49
Yazdabad	35	131	0.22	0.04	0.05	1.38

[a] Discordant Station

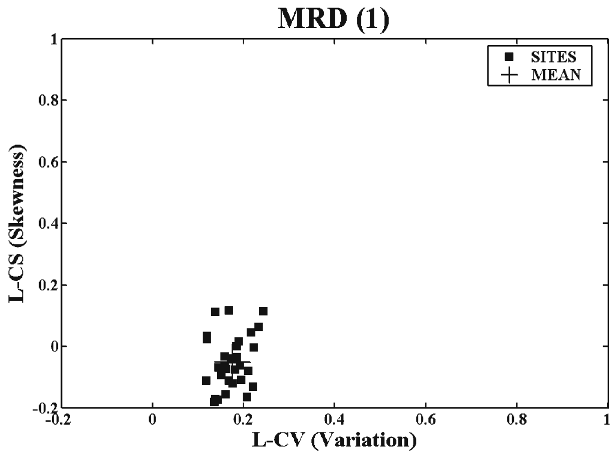
widely scattered around the mean value in the LCv-LCs diagram (Fig. 2). The distribution of LCs and L-Ck is also presented in Fig. 3. Trying to give a general conclusion on the utility of L-moment ratio diagrams to find a parent distribution, Peel et al. (2001) showed that for homogeneous regions, the sample mean is generally a useful variable for distribution selection. In Fig. 3, the sample average is approximately on GLOG distribution and suggests it in the regional frequency analysis (RFA). However, we use goodness-of-fit test for selecting the best regional distribution.

4 Statistical Measures Used in Pooled Frequency Analysis

4.1 Discordancy Measure

The discordancy measure, D_i, in terms of the sample L-moment ratios (L-CV, L-skewness, L-kurtosis) of the at-site data is widely recommended in the screening

Fig. 2 L-Cv–L-Cs moment ratio diagram for 31 AMDSL series in Isfahan Province



process of a typical site from the pooling group. D_i is defined as follows (Hosking and Wallis 1997):

$$D_i = \frac{1}{3}\left(u_i - \bar{u}\right)^T S^{-1}\left(u_i - \bar{u}\right) \tag{7}$$

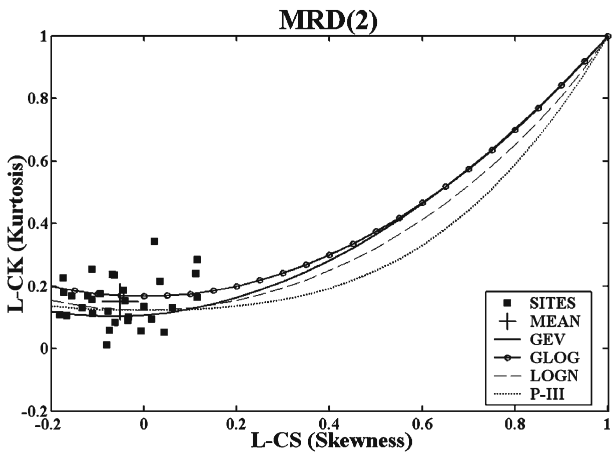
where u_i is the vector of L-moments, LCv, LCs and LCk, for a site i;

$$S = \left(N_s - 1\right)^{-1}\sum_{i=1}^{N_s}\left(u_i - \bar{u}\right)\left(u_i - \bar{u}\right)^T \tag{8}$$

$$\bar{u} = N_s^{-1}\sum_{i=1}^{N_s} u_i \tag{9}$$

where N_S is the number of sites in the group. In general, a site is declared discordant if $Di > 3$.

Fig. 3 L-Cs–L-Ck moment ratio diagram for 31 AMDSL series in Isfahan Province



The study area was checked for discordant stations by the use of Di statistics. The discordancy measures together with the sample L-moments ratios for 31 stations in Isfahan province are given in Table 1. The critical value, 3, is exceeded at only one station, Kalbali, with discordancy measure of 3.46. Thus, this station is excluded from the regional frequency analysis.

4.2 Heterogeneity Measure

The heterogeneity measure estimates the degree of heterogeneity in a group of sites and is used to assess whether the group might reasonably be treated as a homogeneous region. This measure compares the variability of L-moment ratios for the sites in a group with the expected variability which is obtained from simulation for a collection of sites with the same record lengths as those in the group. The homogeneity is assessed at three different levels by focusing on three different measures of dispersion for different orders of the sample L-moment ratios (Hosking and Wallis 1997). The statistics used for homogeneity test are three heterogeneity measures (H) namely H_1, H_2 and H_3 with respect to L-Cv scatter, LCv-LCs and LCv-LCk, respectively. A region is homogenous if any of H_i is less than 1, possibly heterogeneous if H_i is between 1 and 2, and definitely heterogeneous if H_i is greater than 2 (Hosking and Wallis 1997).

Hosking and Wallis (1997) observed that the statistics H_2 and H_3 lack the power to discriminate between homogeneous and heterogeneous regions and that H_1 based on LCv has much better discriminating power. Therefore, the H_1 statistic is recommended as a principal indicator of heterogeneity.

For our study region, the homogeneity measures, H_1, H_2 and H_3 are 0.41, 0.83 and −0.46, respectively, indicating that the study region, Isfahan Province, demonstrates acceptable homogeneity.

4.3 Goodness-of-Fit Measure

The goodness of fit measure is used to identify the regional distribution for the group. The quality of fit is judged by the difference between the regional average $\bar{t}_4$ and the value of τ_4^{Dist} for the fitted distribution. The statistic Z^{Dist} for a chosen distribution is as follows (Hosking and Wallis 1997):

$$Z^{\mathrm{Dist}} = \frac{\bar{t}_4 - \tau_4^{\mathrm{Dist}}}{\sigma_4} \tag{10}$$

where $\bar{t}_4$ is the average L-kurtosis value computed from the data of a given region; τ_4^{Dist} is the average L-kurtosis value computed from simulation for a fitted distribution; and σ_4 is the standard deviation of L-kurtosis values (from simulation).

A given distribution is declared a good fit if $|Z^{\mathrm{Dist}}| \leq 1.64$. If more than one distribution meets the above criterion, the preferred distribution is the one that has the minimum $|Z^{\mathrm{Dist}}|$ value.

In this study, the candidate distributions are Generalized Extreme Values (GEV), Generalized Logistic (GLOG), three-parameter Log Normal (LOGN), Pearson Type III (PIII) and the Generalized Pareto (GP). The $|Z^{\mathrm{Dist}}|$ values for the above distributions are −4.98, −0.47, −3.65, −3.70 and −12.95, respectively. It can be seen that none of the candidates are acceptable except the GLOG distribution function.

5 Multivariate Statistical Analysis

Although the results of L-moment method showed that the entire province is a homogeneous region, it would be reasonable to check if there are smaller homogeneous regions in the province because it would be helpful for watershed managers and agricultural planners to define local characteristics of AMDSL. Thus, we apply multivariate techniques to find smaller groups in the region.

Multivariate techniques have occasionally been used to form groups of similar sites. This kind of group formation is usually based on two types of variables called *at-site statistics* and *site characteristics* (Hosking and Wallis 1997). In hydrological applications the site characteristics would typically include geographical location of the site or other physical properties such as watershed area and slope, associated with the site (e.g. Bates et al. 1998; Chiang et al. 2002). For AMDSL time series, the latitude and longitude of the stations are considered as the site characteristics. The correlation coefficient between stations' coordinates and AMDSL showed a positive significant relationship between AMDSL and station's longitude but insignificant relationship with station's latitude. Thus, for multivariate-based grouping, at-site characteristics are applied. Among different multivariate methods, two methods are more common, *Principal Component Analysis* (PCA) and *Cluster Analysis* (CA). In multivariate analysis, one should make a matrix of the variables observed at each site. In this study, the $(n \times p)$ matrix contains of n at-site statistics and $p = 31$ stations. The correlation matrix of at-site statistics is given in Table 2.

From this table it can be seen that there is a strong relationship between the mean AMDSL and the station's latitude. In other words, the mean AMDSL increases eastward to the arid and semi arid region of Isfahan province. This implies that the mean AMDSL depends on the weather type of the region while the maximum

Table 2 Correlation matrix of selected at-site statistics of AMDSL

	Longitude	Latitude	Mean	Maximum	STDEV	Cv	Cs	Ck	L-Cv	L-Cs	L-Ck
Longitude	1										
Latitude	0.28	1									
Mean	0.60[b]	−0.03	1								
Maximum	0.26	0.11	0.52[b]	1							
STDEV	0.12	0.07	0.16	0.79[b]	1						
Cv	−0.21	0.04	−0.34	0.05	0.38[b]	1					
Cs	0.01	0.16	−0.01	0.69[b]	0.52[b]	−0.08	1				
Ck	−0.08	0.01	0.30	0.78[b]	0.51[b]	0.12	0.55[b]	1			
LCv	−0.26	0.04	−0.58[b]	0.19	0.67[b]	0.59[b]	0.37[a]	0.06	1		
LCs	0.25	0.22	0.12	0.58[b]	0.36[a]	−0.06	0.88[b]	0.34	0.22	1	
LCk	0.20	−0.05	0.60[b]	0.51[b]	0.09	−0.04	0.14	0.72[b]	−0.45[a]	0.15	1

[a]Significant at 5% level

[b]Significant at 1% level

STDEV standard deviation, *Cv* Coefficient of variation, *Cs* Coefficient of skewness, *Ck* Coefficient of kurtosis

AMDSL does not depend on geographical position of the station. It is also important to note that the at-site statistics do not depend on the geographical position of the stations either which indicates that the climate type is not a significant factor on spatial variation of the higher order statistics of AMDSL. However, the maximum observed AMDSL is strongly related to the higher order statistics (Cs, Ck, LCs and LCk). The importance of these significant correlation coefficients will be discussed in the next sections.

5.1 Principal Component Analysis

This method of analysis transforms an original set of variables to a new set of uncorrelated variables called "principal component" (Kaufman and Rousseuw 1990). The first component is a linear combination of the original variables that captures as much of the variation in the original data as possible. The second component captures the maximum variability that is uncorrelated with the first component, and so on.

For observed at-site statistics of 31 AMDSL series, the first three components give reasonable summary of the transformed at-site statistics as they account for 90.7% of its variance. Table 3 lists the variable loading for the first four components derived after VARMAX rotation. For the first principal component, there is a strong positive loading on L-Cs, Cs and relatively strong loading on maximum AMDSL. This component appears to measure the importance of coefficient of skewness for grouping AMDSL in the region. The maximum AMDSL has also a relatively high positive loading. Hence, the magnitude of AMDSL differs from station to station and one can divide the stations based on AMDSL magnitude itself. This component accounts for 38.88% of the variance.

The second component has a strong positive loading on Ck and L-Ck. This component which accounts for 27.36% of variance appears to show the role of kurtosis coefficient in grouping the AMDSL time series and their distribution functions in Isfahan province. This component has a relatively strong loading on maximum and mean AMDSL too.

The third component accounts for 13.36% of the variance between AMDSL time series and could be interpreted as the contrast between Cv, L-Cv, standard deviation and Ck (second component). The forth component describes 11.09% of the variance between stations which are due to geographical position of the stations.

Table 3 Principal component loading

Variables	Component			
	1	2	3	4
Longitude	0.098	−0.045	−0.070	0.932
Latitude	−0.19	0.20	0.08	0.63
Mean	−0.010	0.535	−0.233	0.755
Maximum	0.599	0.609	0.336	0.324
STDEV	0.432	0.247	0.778	0.232
Cv	−0.221	0.054	0.827	−0.206
Cs	0.965	0.181	0.112	−0.083
Ck	0.371	0.867	0.200	−0.108
LCv	0.332	−0.312	0.828	−0.272
LCs	0.912	0.065	0.008	0.142
LCk	−0.001	0.916	−0.158	0.177

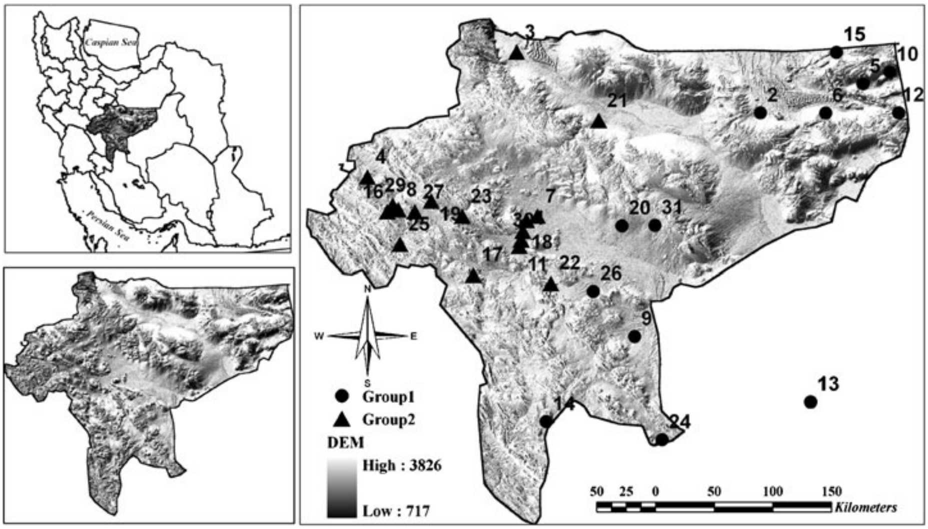


Fig. 4 Location map of two western (*triangles*) and eastern (*circles*) sub regions

5.2 Cluster Analysis

The aim of cluster analysis is to assign objects to a set of exclusive groups/clusters with maximum similarity of the group members and dissimilarity of the groups.

For cluster analysis, Ward's minimum variance method (Kaufman and Rousseuw 1990) and the dissimilarity squared Euclidean distance measure were used in this study. Our choice incorporates the agglomeration methods and distance metrics that Nathan and McMahon (1990) found to be most successful for the regionalization in southern Australia, Ramos (2001) suggested them for regionalization of Mediterranean rainfall and Modarres (2006) used them for grouping Iranian precipitation climates.

To select the suitable number of clusters, the R^2 criterion was applied. In each step of clustering, R^2 explains the variance between derived clusters. In the present study, two clusters explain 88.07% of the variance between two groups. The geographical distribution of these two groups is shown in Fig. 4. The stations of the first group are located in the western region of Isfahan province while the stations of the second group are located in eastern and southeastern regions of the province.

Then we apply L-moments method in each group to check the homogeneity and select the regional frequency distribution for each region separately.

The homogeneity measures and the $|Z^{Dist}|$ values for different distributions are given in Tables 4 and 5. According to the homogeneity measure, H_1, which is more important for homogeneity test (Hosking and Wallis 1997), two sub regions

Table 4 The homogeneity measures for two sub regions

Sub region	H_1	H_2	H_3
West	0.2	−1.43	−1.56
East	0.14	−1.24	−1.41

Table 5 Goodness-of-fit-test, $|Z^{Dist}|$, for two sub regions

Sub region	GLOG	GEV	LN3	P3
West	0.39[a]	−3.11	−2.02	−2.08
East	2.69	−0.7[a]	0.33[a]	0.29[a]

[a]The distribution may be accepted as a regional distribution

demonstrate acceptable homogeneity. From Table 5, it can be seen that none of the candidates are acceptable for the western region except GLOG while, for the eastern region, all the candidates are acceptable except GLOG. However, the Pearson Type III (PIII) distribution has the smallest $|Z^{Dist}|$ value and is selected as the regional frequency distribution for the eastern region.

6 Statistical Comparison of the Groups

Although the entire region of Isfahan province was shown to be homogeneous using L-moments method but in the previous section, we showed that there exist two groups of AMDSL in Isfahan Province. In this section, we check whether or not these two groups are actually different from each other. For this purpose, we apply three non-parametric tests in the following sections. These methods check if the means, variances and the cumulative distribution functions (CDFs) of the two groups are equal or not. These methods are also used to evaluate different hydrologic models (Modarres 2009).

Parametric and Non-Parametric Test for the Difference of Two Population Means Parametric T test is applied to compare the mean of two populations. However, for robustness, a non-parametric test is also applied. One of the best non-parametric methods for constructing a hypothesis test *p*-value for μ_1–μ_2 (difference of two population means) is the Wilcoxon rank sum method (Conover 1980). This method is used in the present work to test the difference of the means of two dry spell groups in west and east of Isfahan province. At any significance level greater than the *p*-value, one rejects the null hypothesis, and at any significance level less than the *p*-value, one accepts the null hypothesis. For example, if *p*-value is 0.04, one rejects the null hypothesis at a significance level of 0.05, and accepts the null hypothesis at a significance level of 0.01. The null hypothesis of Wilcoxon test can be defined at:

$$H_0 : \mu_1 - \mu_2 = 0 \tag{11}$$

$$H_a : \mu_1 - \mu_2 \neq 0 \tag{12}$$

Non-Parametric Test for the Equality of Two Population Variances The equality of two population variances can be tested using Levene's test. The hypothesis for the Levene's test can be defined as (Levene 1960):

$$H_0 : \sigma_1 = \sigma_2 = \ldots = \sigma_\mathrm{k} \tag{13}$$

$$H_a : \sigma_1 \neq \sigma_\mathrm{j} \neq \ldots \neq \sigma_\mathrm{k} \qquad \text{for at least one pair } (i,\ j) \tag{14}$$

In performing Levene's test, a variable X with sample size N is divided into k sub groups, where Ni is the sample size of the ith sub group, and the Levene test statistic is defined as:

$$W = \frac{(N-k)\sum_{i=1}^{k} N_i\left(\bar{Z}_i - \bar{Z}\right)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{N_i}\left(Z_{ij} - \bar{Z}_i\right)^2}$$

where Z_{ij} is defined as:

$$Z_{ij} = \left|X_{ij} - \bar{X}_i\right| \tag{15}$$

where $\bar{X}_i$ is the median of the ith sub group, $\bar{Z}_i$ are the group means of the Z_{ij} and Z_i is the overall mean of the Z_{ij}. The Levene's test rejects the hypothesis that the variances are equal if

$$W > F_{(\alpha,k-1,N-k)}$$

where $W > F_{(\alpha,k-1,N-k)}$ is the upper critical value of the F distribution with $k-1$ and $N-k$ degrees of freedom at a significant level of α.

Non-Parametric Test for Equality of CDFs of Two Populations Kolmogorov–Smirnov non-parametric test (Yevjevich 1972) is used to compare cumulative distribution function (cdf) of two groups of dry spell regions. Suppose, $F_1(x)$ and $F_2(x)$ are two cdfs of two samples data of a variable x. The null hypothesis and the alternative hypothesis concerning their cdfs are:

$$\begin{aligned} H_0 &: F_1(x) = F_2(x) \quad \text{for all } x \\ H_a &: F_1(x) \neq F_2(x) \quad \text{for at least one value of } x \end{aligned}$$

and the test statistics, Z is defined as:

$$Z = \sup_x \left|F_1(x) - F_2(x)\right| \tag{16}$$

which is the maximum vertical distance between the distributions $F_1(x)$ and $F_2(x)$. If the test statistic is greater than the critical value, the null hypothesis is rejected.

In the present study, the above methods are applied to compare mean, maximum, standard deviation, statistical moments and L-moments of AMDSL of two west and east regions derived from L-moment method in the previous section. The results are given in Table 6. In this Table, the bold values show that the null hypothesis is rejected.

It is clear that the maximum and mean AMDSL are significantly different in two regions as the p-value of Wilcoxon and T test's statistics reject the null hypothesis of equality of the mean, μ, at 1% significant level. In other words, the multivariate cluster analysis is an appropriate tool for grouping the AMDSL time series according to their magnitude. The Kolmogorov–Smirnov (K–S) test also indicates that the probability distribution of the mean and AMDSL time series are significantly different in two regions. The Levene's test indicates that the variance of mean and maximum AMDSL of western region can be considered equal with those of the eastern region. However, the Wilcoxon rank sum test shows a possible, but not significant at 95% level, difference between the standard deviation of the AMDSL

Table 6 Results of parametric and non-parametric tests for equality of AMDSL in west and east regions

Variable	Wilcoxon test		K–S test		T test		Levene's test	
	W	*p-value*	Z	*p-value*	Z	*p-value*	W	*p-value*
Maximum	***−2.810***	***0.005***	***0.580***	***0.005***	***−3.550***	***0.009***	0.897	*0.351*
Mean	***−3.410***	***0.006***	***0.620***	***0.003***	***−3.780***	***0.007***	0.648	*0.428*
lcv	0.820	*0.410*	0.160	*0.960*	1.006	*0.320*	0.141	*0.710*
lcs	***−2.080***	***0.037***	***0.470***	***0.041***	***−2.110***	***0.042***	2.310	0.140
lck	−1.610	*0.110*	0.430	*0.080*	−1.270	*0.210*	0.168	*0.680*
stdev	−1.410	*0.077*	0.440	*0.070*	−1.088	*0.280*	0.670	*0.420*
cv	1.150	*0.250*	0.320	*0.290*	0.670	*0.500*	0.008	*0.930*
cs	−0.770	*0.440*	0.250	*0.600*	−0.410	*0.680*	0.004	*0.940*
ck	−0.750	*0.450*	0.230	*0.730*	0.100	*0.910*	0.460	*0.500*

time series of western and eastern region. It should also be noted that no significant difference is observed for other statistics of AMDSL time series. However, the Wilcoxon rank sum test and T test reject the null hypothesis of the equality of the mean of L-Cs of the regions. The cdfs of L-Cs of the two regions are also different because the K–S test rejects the null hypothesis at 95% level. The results of K–S test on L-Ck shows a minor difference at 90% significant level ($0.05 < \text{p-value} < 0.1$). This is the main reason that the regional frequency distribution is different in two regions according to the goodness-of-fit-test of L-moment method (Table 4). As mentioned by Hosking and Wallis (1997), the goodness-of-fit-test is judged by how well the L-skewness and L-kurtosis of the fitted distribution match the regional average L-skewness and L-kurtosis of the observed data. The above results confirm the results of PCA which illustrates higher loadings of L-Cs and L-Ck in the firs and second component (Table 3).

It is also important to note that the L-Cv does not show any significant difference because at-site sample L-Cv values are used as the basis for forming homogenous region. In other words, the equality of L-Cv values of the two regions show a similar degree of homogeneity of western and eastern regions and does not affect the type of regional frequency distribution fitted to each group.

The box plots of L-moments and ordinary moments of AMDSL of western and eastern regions are illustrated in Figs. 5 and 6. These figures verify the results

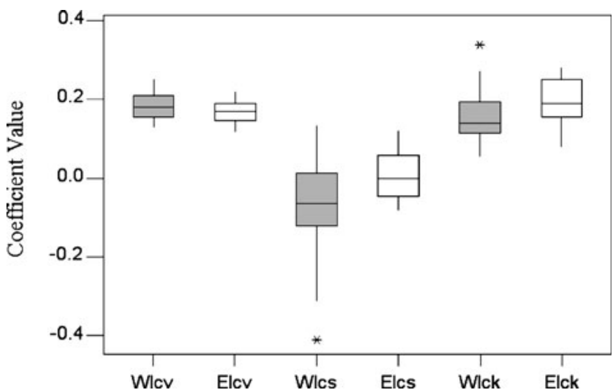


Fig. 5 Box plots of L-moments (*Lcv*, *Lcs* and *Lck*) of AMDSL in western (*W*) and eastern (*E*) region of Isfahan province

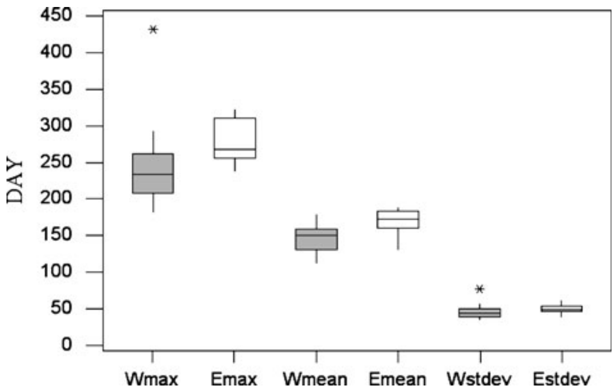


Fig. 6 Box plots of maximum (*max*), mean and standard deviation (*stdev*) of AMDSL in western (*W*) and eastern (*E*) region of Isfahan province

of Table 6 where the difference between L-Cs of the two regions is statistically significant. They also show a very small different of L-Ck and major significant of the maximum and mean AMDSL of the two regions in eastern semi arid region and western sub humid region.

7 Conclusions

Our exploratory study of regional annual maximum dry spell length time series was applied for regional frequency analysis of extreme dry events in Isfahan Province where GLOG distribution is found to be the best regional frequency distribution. Despite the homogeneity of the entire province deduced from the application of L-moment method, the application of cluster analysis indicated the presence of two homogeneous regions that were shown to be homogeneous based on homogeneity measure and *H* statistic. The use of L-moment, PCA and CA indicated the following main results:

1. The assignment of AMDSL series to regions on the basis of geographical location showed that the arid and semi arid regions are typically different from the more humid regions of Isfahan Province and the eastern semi arid region has a relatively higher AMDSL than the western region. The results of clustering technique and nonparametric test verify this spatial variation of AMDSL.
2. The delineation of AMDSL regions on the basis of sample L-Cv would reflect only the pattern of noise in the data and have no physical significance. The correlation coefficient between L-Cv and the latitude and longitude of the stations as the only physical properties of the stations (Table 2) and the nonparametric tests reveal that the L-Cv is statistically equal in two homogenous regions in west and east of the province. Therefore, the coefficient of variation (Cv) and L-coefficient of variations (L-Cv) stand in the third principal component.
3. For Isfahan Province and on the basis on PCA, the L-coefficient of skewness (L-Cs) and the maximum AMDSL are the most important factors describing the dissimilarity between the stations and the AMDSL groups. The use of nonparametric tests also confirmed the significant difference between the two

groups according to LCs and maximum AMDSL of the groups. The L-Cs values of the western region are relatively higher than the eastern semi arid region.
4. For Isfahan Province, the L-Ck was also found to describe a main proportion of the variation of the AMDSL according to PCA. Together with LCs, L-Ck is the main variable used for goodness-of-fit-test of a regional frequency distribution.

In general, this study showed that it may be useful to apply multivariate techniques together with L-moment in hydrologic regionalization when at-site statistics are available with at-site characteristics. In other words multivariate methods could be used to make a preliminary determination of group membership for regional frequency analysis using L-moments. It should also be noted that the generality of the findings of this study for the annual maximum dry spell length needs to be tested in other geographical regions with different climates and different dry spells thresholds.

Acknowledgements The author is grateful to Mr. Ali Sarhadi to provide the Digital Elevation Model (DEM) of Isfahan province. The comments of the two anonymous reviewers are also acknowledged.

References

ARIDE (1999) Methods for regional classification of streamflow drought series: the EOF method and L-moments. Technical report, no, 2

Bates BC, Rahman A, Mein G, Weinmann PE (1998) Climate and physical factors that influence the homogeneity of regional floods in southern Australia. Water Resour Res 34:3369–3381

Burn DH (1990) An appraisal of the “region of influence” approach to flood frequency analysis. Hydrol Sci J 35:149–165

Byun HR, Wilhite DA (1999) Objective quantification of drought severity and duration. Am Meteorol Soc 12:2747–2756

Cavadias GS, Ouarda TBMJ, Bobee B, Girard C (2001) A canonical correlation approach to the determination of homogeneous regions for regional flood estimation of ungauged basins. Hydrol Sci J 46:499–512

Chen YD, Huang G, Shao Q, Xu CY (2006) Regional analysis of low flow using L-moments for Dongjiang basin, South China. Hydrol Sci J 51:1051–1064

Chiang SM, Tsay TK, Nix SJ (2002) Hydrologic regionalization of Watersheds. I. Methodology development. J Water Resour Plan Manage 128:4–20

Conover WJ (1980) Practical nonparametric statistics, 2nd edn. Wiley, New York

Dracup JA, Lee KS, Paulson EN (1980) On the statistical characteristics of drought events. Water Resour Res 16:289–296

Durrans SR, Tomic S (1996) Regionalization of low-flow frequency estimations: an Alabama case study. Water Resour Bull 32:23–37

Guttman NB, Wallis JR, Hosking JRM (1992) Spatial comparability of the Palmer drought severity index. Water Resour Bull 28:1111–1119

Hosking JRM, Wallis JR (1997) Regional frequency analysis: an approach based on L-moments. Cambridge University Press, Cambridge

Kaufman L, Rousseuw PJ (1990) Finding groups in data: an introduction to cluster analysis. Wiley, New York, 344 pp

Kjeldsen TR, Smithers JC, Schulze RE (2002) Regional flood frequency analysis in the Kwazulu-Natal province, South Africa, using the index-flood method. J Hydrol 255:194–211

Kroll CK, Vogel RM (2002) Probability distribution of low streamflow series in the United States. J Hydrol Eng 7:137–146

Lana X, Burgueno A (1998a) Daily dry-wet behavior in Catalonia (NE Spain) from the viewpoint of the first and second order Markov chains. Int J Clim 18:793–815

Lana X, Burgueno A (1998b) Probabilities of repeated long dry episodes based on the Poisson distribution. An example for catalonia (NE Spain). Theor Appl Clim 60:111–120

Lana X, Martinez MD, Burgueno A, Serra C, Martin-Vide J, Gomez L (2006) Distribution of long dry spells in the Iberian peninsula, years 1951–1990. Int J Clim 26:1999–2021
Levene H (1960) Contributions to probability and statistics. Stanford University Press, Palo Alto
Lim YH, Lye LM (2003) Regional flood estimation for ungauged basins in Sarawak, Malaysia. Hydrol Sci J 48:79–94
Martin-Vide J, Gomez L (1999) Regionalization of peninsular Spain based on the length of dry spells. Int J Clim 19:537–555
Modarres R (2006) Regional precipitation climates of Iran. J Hydrol (NZ) 45:13–27
Modarres R (2008) Regional frequency distribution type of low flow in north of Iran by L-moments. Water Resour Manage 22:823–841
Modarres R (2009) Multi-criteria validation of artificial neural network rainfall-runoff Modeling. Hydrol Earth Sys Sci 13:411–421
Modarres R, da Silva VPR (2007) Rainfall trends in arid and arid regions of Iran. J Arid Environ 70:344–355
Modarres R, Sarhadi A (2009) Rainfall trends analysis of Iran in the last half of the twentieth century. J Geophys Res 114:D03101. doi:10.1029/2008JD010707
Nathan RJ, McMahon TA (1990) Identification of homogeneous regions for the purpose of regionalization. J Hydrol 121:217–238
Parida BP, Kachroo RK, Shrestha DB (1998) Regional flood frequency analysis of Mahi-Sabarmati Basin (Sub zone 3-a) using index flood procedure with L-moments. Water Resour Manage 12:1–12
Peel MC, Wang QJ, Vogel RM, McMahon TA (2001) The utility of Lmoment ratio diagrams for selecting a regional probability distribution. Hydrol Sci J 46:147–155
Ramos MC (2001) Divisive and hierarchical clustering techniques to analyze variability of rainfall distribution patterns in a Mediterranean region. J Hydrol 57:123138
Reed DW, Jacob D, Robson AJ, Faulkner DS, Stewart EJ (1999) Regional frequency analysis: a new vocabulary. In: Gottschalk L, Olivry J-C, Reed D, Rosbjerg D (eds) Hydrological extremes: understanding, predicting, mitigating. In: Proceedings of the Birmingham Symposium July 1999, IAHS, Idyllwild, pp 237–243
Stedinger JR, Vogel RM, Foufoula-Georgiou E (1993) Frequency analysis of extreme events. In: Maidment DR (ed) HandBook of hydrology. McGraw Hill, New York, pp 18.1–18.66
Vicente-Serrano SM, Begueria-Portugues S (2003) Estimating extreme dry-spell risk in the middle Ebro vally (NE Spain): a comparative analysis pf partial duration series with general Pareto distribution and annual maxima series with Gumbel distribution. Int J Clim 23:1103–1118
Vogel RM, Fennessey NM (1993) L-moment diagram should replace product moment diagram. Water Resour Res 29:1745–1752
Vogel RM, Wilson I (1996) Probability distribution of annual maximum, mean, and minimum streamflows in the United States. J Hydrol Eng 1:69–76
Wallis JR, Schaefer MG, Barker BL Taylor GH (2007) Regional precipitation-frequency analysis and spatial mapping for 24-hour and 2-hour durations for Washington state. Hydrol Earth Sys Sci 11:415–442
Yevjevich V (1972) Probability and statistics in hydrology. Water Resources, Highlands Ranch, 386 pp