Low Flow Scaling with Respect to Drainage Area and Precipitation in Northern Iran

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Abstract: Low flow spatial scaling relationships have been defined by log-log linearity between 7-, 15-, 30-, and 60-day low flow probability weighted moments (PWMs) and drainage area size in north of Iran. The PWMs are used to avoid the influence of outliers. Across the entire region, the regression relationship is not significant which is believed to be due to climate heterogeneity of the region. Dividing the region into two humid and semiarid regions, the log-log relationship is found to be significant for the humid subdivision while it is not significant in semiarid region. This implies that in a heterogeneous climate regime, scaling alone is a poor method for extending low flow at-site probabilistic behavior to a region. However, for the semiarid subdivision, the relationship between mean annual rainfall and PWMs verges on significant which suggests to the hydrologist that other alternatives to drainage area size should be examined for scaling low flows in such regions.

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Introduction

The statistical properties of low flows are required for different purposes or the effective use of freshwater. However, these statistical properties are usually required at sites or regions where low flow data are not available or are not long enough to estimate statistical properties.

The regional analysis of low flow is often carried out to estimate these properties. Since Dalrymple (1960) introduced the index flood method, a number of methods have been proposed for flood and low flow regional analysis. Most of these methods consider physical properties of the basins for regionalization and construct empirical models (Pandey 1998). In recent years, the scaling concept provides a new scope in modeling and investigating spatial variability of different hydrologic phenomenon, e.g., floods and low flows, which are governed by different physiographical and climatic factors.

Since Gupta and Waymire (1990) and Smith (1992) assessed the scaling behavior of a regional flood series, numerous studies have investigated the spatial scaling behavior of flood peaks [e.g., Gupta et al. (1994, 1996); Gupta and Dawdy (1995); Pandey (1998); Ogden and Dawdy (2003); Eaton et al. (2002); and Mantilla et al. (2006)]. Some studies have also investigated rainfall scaling [e.g., Schaefer (1990); Kumar and Foufoula-Georgiou (1993); Over and Gupta (1996), and Burlando and Rosso (1996)]. However, the scale of low flows has not been widely investigated. The generating mechanism of low flows is dramatically different from that of flood peaks (Furey and Gupta 2000). As mentioned above, low flow statistics are important for the effective use of freshwater resources. The objective of this study is to examine the scale of low flows in northern Iran. A minor objective of this study is to determine whether or not the scaling concept is valid in semiarid region.

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Spatial Scaling of Low Flows

To show low flow scaling, we follow the methods presented by Vogel and Sankarasubramanian (2000) and Yue and Wang (2004). If we let the time series of annual minimum mean flow of two basins with drainage areas A_i and A_j be given by Q_i and Q_j, the scaling between these two basins is described by

$$Q_i \overset{d}{=} h(A_i,\ A_j) Q_j \tag{1}$$

where $\overset{d}{=}$ implies equality of the probability distributions associated with the low flows and $h(A_i,\ A_j)$=scale transformation function. Based on simple scaling laws (Gupta and Waymire 1990), we have

$$E[Q_i^k(A_i)] = \left(\frac{A_i}{A_J}\right)^{k\theta} E[Q_j^k(A_j)] \tag{2}$$

where k stands for the order of a moment and $k\theta = H$. The scaling behavior of low flows could be investigated by two types of moments, product moments (PMs) and probability weighted moments (PWMs). In this study, the PWMs are used because the main advantage of PWMs over PMs is the robust behavior of PWMs to outliers (Pandey 1998; Yue and Wang 2004).

Low Flow Scaling Based on PWMs

The scaling behavior of low flows can be assessed based on the log relationship between drainage area and probability weighted

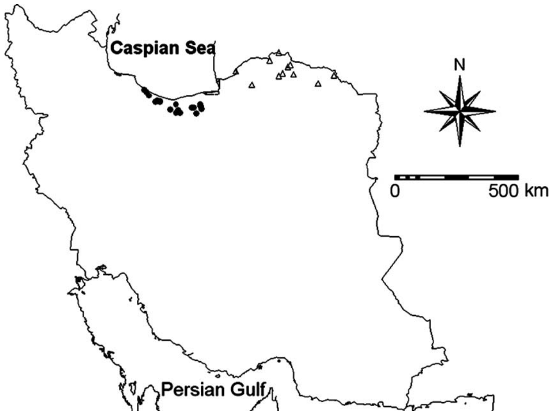

Fig. 1. Distribution of sample gauges in North of Iran. Black cycles show humid region and triangles show the semiarid region.

moments of different orders (Kumar et al. 1994). Using PWMs of low flows, taking the basin j as the reference basin with the area equal to unit and taking the natural logarithm of Eq. (2) give the following equation:

$$\ln(\mathrm{PWM}^k_{A_i}) = \ln(\mathrm{PWM}^k_{A_j}) + H \ln(A_i/A_j) \quad (3)$$

in which H=scaling exponent. By taking the reference area A_j =1, the above equation can be rewritten as

$$\ln(\mathrm{PWM}^k_{A_i}) = c_k + H \ln(A_i) \quad (4)$$

Low flow can then be presented by simple scaling concepts if Eq. (4) holds. In other words, low flows appear to obey a simple scaling law if the logarithm of the k-order moment of the streamflow is a linear function of the logarithm of drainage area and its slope is a linear function of the moment order (k).

The kth PWM of the annual streamflow of basin i is

$$(\mathrm{PWM}^k_{A_i}) = \int_0^\infty q_i F^k_{Q(A_i)}(q_i) dF_{Q(A_i)} q_i \quad (5)$$

in which $F(q_i)=\Pr[Q^3 q_i]$. The $\mathrm{PWM}^k_{A_i}$ can be estimated from the sample data (Stedinger et al. 1993).

Database

In this study, the annual minimum average 7-, 15-, 30-, and 60-day streamflows of 30 watersheds in northern Iran (Fig. 1) for the period 1960–2000 are selected for scaling. The region is divided into two regions based on climatic characteristics (Modarres 2008). The mean annual rainfall (MAR) in the western region is 575 mm while the MAR in the eastern region is 279 mm. The area of the selected watersheds varies between 104 to 2,709 km^2 in the western region while in the eastern drainage area it varies between 114 to 20,700 km^2. The autocorrelation of low flow data set was investigated by the use of autocorrelation function and no significant correlation was found (Modarres 2008). Although the effect of spatial correlation on low flow scaling has not been fully investigated, we can say that the spatial correlation is not significant in northern Iran using the Pearson correlation coefficient.

Relationship between Probability Weighted Moments and Watershed Area

The assessment of low flow scaling is carried out in this section by fitting a linear regression between the natural logarithm of estimated PWMs of selected low flow series and the natural logarithm of watershed area. To assess the effect of climate heterogeneity on low flow scaling, we examine the scaling in two regions, western (humid) and eastern (semiarid). In other words, we consider the whole region for scaling first; then, low flow scaling is investigated in humid and semiarid regions, separately.

Low Flow Scaling for Entire Region

Fitting log-log regression between PWMs of order $k=1$ to $k=10$ and the drainage area of the watershed shows that there is no significant relationship between the natural logarithm of PWMs and the natural logarithm of the watershed area. The coefficient of determination (R^2) for the regression equations (not shown here) differs, on average, from 0.22 to 0.47 for 7- and 60-day low flows, respectively. This suggests that in a region with heterogeneous climate, scale alone is a poor predictor of the PWMs of low flows. Vogel and Sankarasubramanian (2000) also concluded that for the Upper Colorado and Missouri river basins, the scaling of annual flow is not meaningful due to the heterogeneous climate. Comparing the scaling behavior of low flows in subclimatic regions of Canada, Yue and Wang (2004) also demonstrated that the coefficients of determination (R^2) are not strong enough to consider the entire country as one completely homogeneous region and scaling exponents are different in each of the subclimatic regions. However, they did not mention any semiarid or arid region in Canada.

Low Flow Scaling for the Humid Region

The log-log regression equation was developed to test the scaling behavior of low flows in the humid region of the study area. On the basis of the PWMs, the estimated linear regression parameters, c and H of Eq. (3), the coefficients of determination (R^2), and the t statistics (t_s) for the first 10 orders of the PWMs of annual mean 7-, 15-, 30-, and 60-day flows are presented in Table 1. It is evident that all R^2 values are significant at the 99% significant level. This indicates that all the identified log-log linear relationships between PWMs and drainage area are statistically significant. To check the validity of the regression equations, the Kolmogrov-Smirnov normal test was applied on the residuals. The results show that the residuals are normally distributed and we cannot reject the null hypothesis of normality at the 5% significant level.

For the purpose of illustration, Fig. 2 shows the goodness of fit of regression lines to the observations of the first two PWMs. Fig. 3 shows H values versus the corresponding PWM orders k. It is clear that H values are almost the same and independent of PWM order k. These results confirm the fact that the annual minimum mean 7-, 15-, 30-, and 60-day flow generally obey the simple scaling law across humid region of the study area.

Low Flow Scaling for the Semiarid Region

The MAR of semiarid region is 279 mm and classified as semiarid climate. The average values of the coefficients of determination (R^2) in Eq. (4) for different orders of PWMs are 0.05, 13, 18, and 0.21 for 7-, 15-, 30-, and 60-day low flows, respectively.

Table 1. Regression Coefficients in Eq. (4) for Annual Minimum 7-, 15-, 30- and 60-Day Flows

	7 days				30 days			
PWM order (k)	c	H	R^2	t_s	c	H	R^2	t_s
1	−4.142[a]	0.585[a]	0.558[a]	4.630	−3.877[a]	0.609[a]	0.663[a]	5.772
2	−4.62[a]	0.597[a]	0.559[a]	4.640	−4.227[a]	0.611[a]	0.669[a]	5.856
3	−4.96[a]	0.604[a]	0.553[a]	4.586	−4.488[a]	0.613[a]	0.670[a]	5.880
4	−5.226[a]	0.607[a]	0.54[a]	4.472	−4.682[a]	0.614[a]	0.671[a]	5.883
5	−5.429[a]	0.608[a]	0.527[a]	4.350	−4.844[a]	0.612[a]	0.664[a]	5.793
6	−5.599[a]	0.609[a]	0.511[a]	4.215	−4.988[a]	0.612[a]	0.660[a]	5.744
7	−6.023[a]	0.612[a]	0.603[a]	5.085	−5.11[a]	0.611[a]	0.654[a]	5.67
8	−6.058[a]	0.615[a]	0.611[a]	5.116	−5.226[a]	0.612[a]	0.65[a]	5.618
9	−6.131[a]	0.617[a]	0.534[a]	4.41	−5.33[a]	0.612[a]	0.646[a]	5.575
10	−6.154[a]	0.619[a]	0.571[a]	4.758	−5.43[a]	0.612[a]	0.640[a]	5.498
	15 days				60 days			
1	−3.961[a]	0.598[a]	0.629[a]	5.368	−3.773[a]	0.621[a]	0.578[a]	4.821
2	−4.327[a]	0.603[a]	0.648[a]	5.595	−4.103[a]	0.62[a]	0.582[a]	4.870
3	−4.601[a]	0.607[a]	0.653[a]	5.661	−4.338[a]	0.618[a]	0.583[a]	4.877
4	−4.848[a]	0.608[a]	0.681[a]	6.02	−4.53[a]	0.618[a]	0.584[a]	4.878
5	−5.011[a]	0.61[a]	0.655[a]	5.686	−4.67[a]	0.614[a]	0.578[a]	4.83
6	−5.191[a]	0.612[a]	0.687[a]	6.111	−4.811[a]	0.614[a]	0.578[a]	4.825
7	−5.33[a]	0.613[a]	0.682[a]	6.035	−4.929[a]	0.613[a]	0.575[a]	4.796
8	−5.439[a]	0.615[a]	0.643[a]	5.533	−5.041[a]	0.613[a]	0.571[a]	4.758
9	−5.599[a]	0.616[a]	0.692[a]	6.183	−5.131[a]	0.611[a]	0.567[a]	4.719
10	−5.669[a]	0.618[a]	0.651[a]	5.635	−5.220[a]	0.61[a]	0.564[a]	4.690

[a]Significant at 1% level.

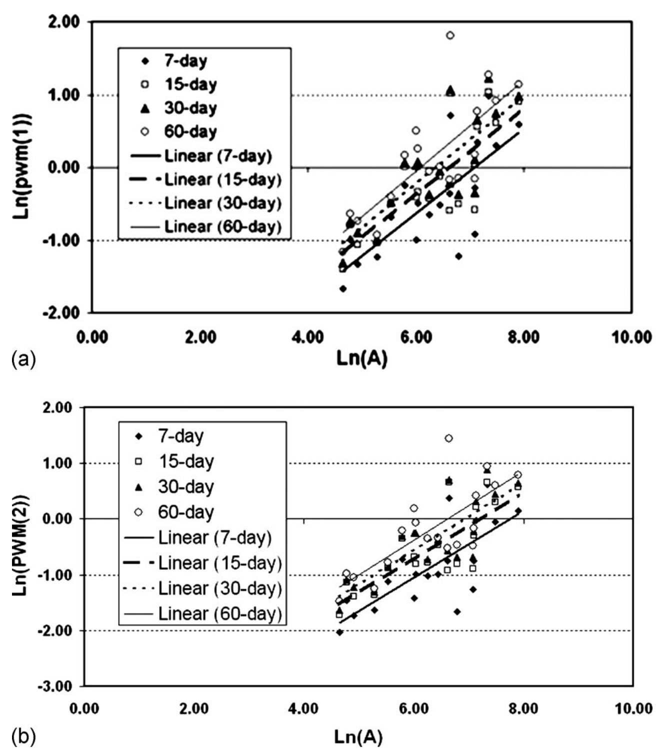

Fig. 2. Log relationship between drainage area for the humid region and the (a) first; (b) second orders of PWMs of annual mean 7-, 15-, 30-, and 60-day flows

These coefficients do not show any significant relationship between PWMs and drainage area in the semiarid region. These insignificant relationships imply that in the semiarid region of the study area, the drainage area is not a good predictor for which to extend the at-site probabilistic properties of low flow across the region. In other words, scaling alone is not a good method for low flow regionalization in semiarid regions of northern Iran.

Alternative Variables for Low Flow Scaling

In this section, we seek an alternative variable that can be used in Eq. (4) instead of the drainage area. Furey and Gupta (2000) investigated the space and time variability of low streamflows in a river network in terms of physics laws pertaining low flows.

They developed an equation which shows the relationship between the square root of the drainage area and the base flow. Replacing $\ln(A)$ with $\sqrt{A}$ in Eq. (4), we get similar results for the entire region, and the humid and semiarid regions. In other words, by adopting Eq. (4) it is not found that it is statistically significant for the entire study area or the semiarid region but is significant for the humid region only.

Furey and Gupta (2000) found that this relationship is weak due to the permeable formations of Flint River basin in Georgia. The semiarid region of our study area consists of soft unconsolidated and semiconsolidated sand, gravel, and lime stone with an extensive network of faults. These geological conditions may affect the relationship between the square root of the drainage area and base flow and make it insignificant for semiarid region and entire region of the study area.

However, the correlation coefficients between the logarithm of PWMs of different orders (k) and the MAR and logarithm of

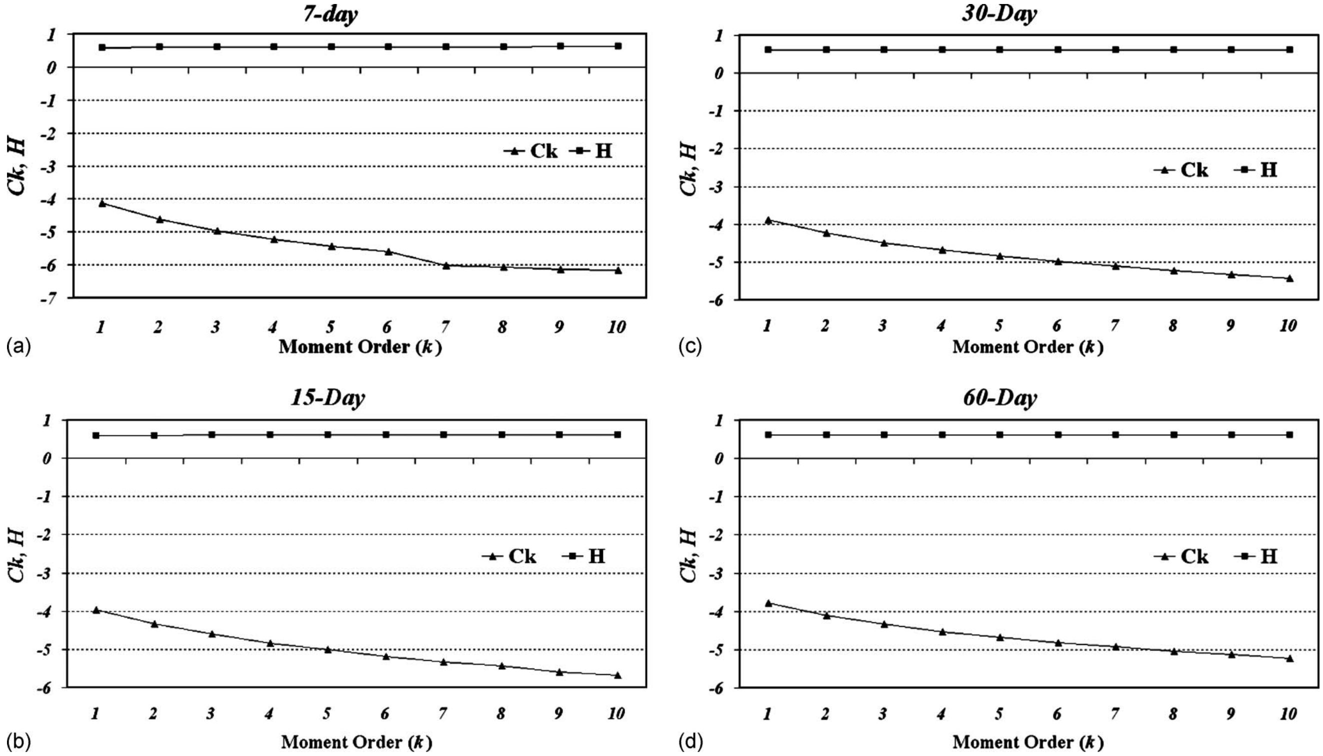

Fig. 3. Plot of the scale exponent (H) versus PWMs order (k) of annual minimum (a) 7-day; (b) 15-day; (c) 30-day; and (d) 60-day flows of Eq. (4) for the humid region

MAR as two other alternative variables for drainage area of the watershed are significant for 15-day low flow PWMs. This may show that in arid and semiarid regions, one can find different alternative variables for extending probabilistic behavior of low flows to a region.

Thus, one can add MAR to Eq. (4) and check whether or not the following equation is significant:

$$\ln(\mathrm{PWM}_{A_i}^{k}) = c_k + H \ln(A_i) + G \ln(P_i) \qquad (6)$$

where P=MAR. Similar to H, one can test if G remains constant when PWM order k increases. If both H and G remain constant, we can conclude that low flows obey simple scaling. In the following subsections, Eq. (6) is applied for humid, semiarid, and the entire regions to investigate the effect of physical and climatic conditions on low flow scaling.

Two Variable Scaling for Entire Region

The regression relationship in the form of Eq. (6) is estimated for 7-, 15-, 30, and 60-day low flows of the entire region. The results show that Eq. (6) is significant for all low flow durations except for 7-day low flows. One can conclude that both H and G coefficients are significant at the 5% level of significant. The K-S test also shows that the null hypothesis of residual normality could not be rejected at 5% significant level.

On the other hand, for 7-day low flows, H is not significant and thus $\ln(A)$ does not enter Eq. (6) while G is significant at 5% level for PWM orders of k=1 to 10 and, therefore, only $\ln(P)$ remains in the equation. This may illustrate that short duration low flows are more sensitive to rainfall which may be the effect of unconsolidated geologic formations of the study area. Fig. 4 shows G values plotted against corresponding PWM orders k. It is clear that G does not differ at different orders of k. The chi-square test applied to test the variance of G for different low flow durations shows that the variance of G is not statistically different from zero. Thus, the simple scaling of low flows is evident.

Two Variable Scaling for Humid Region

The results of regression analysis of Eq. (6) for the humid region showed that G, and thereby, $\ln(P)$, is not significant while H, and thus $\ln(A)$, is significant at 1% level. In other words, Eq. (6) is similar to Eq. (4) which shows that low flows obey simple scaling in the humid region.

Two Variable Scaling for Semiarid Region

Similar to the case for the entire region, Eq. (6) is fitted to the semiarid region's low flows. The results show that moving to a

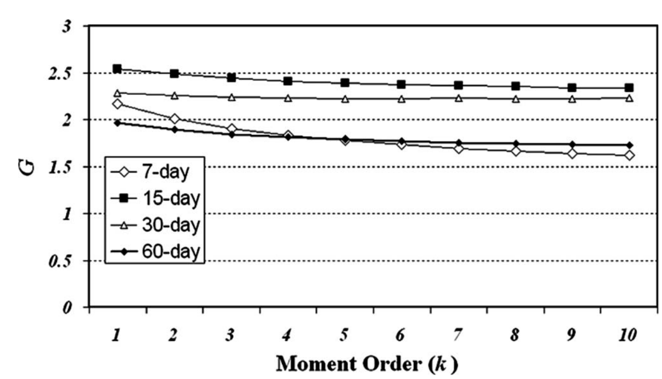

Fig. 4. Plot of the scale exponent (G) versus PWMs order (k) of annual minimum 7-, 15-, 30-, and 60-day flows of Eq. (4) for study area

higher order of PWMs, the coefficient of H is significant only for 30-day low flows while the G coefficient is also significant for 15-day low flows in a similar way but at a weaker level of significant ($0.05 < p < 0.1$). Thus we can conclude that for the semiarid region of the study area, low flows of medium duration are significantly correlated to the drainage area and rainfall total. In other words, low flows of shorter duration respond to rainfall variation while low flows with longer duration vary with the size of the drainage area. It should also be noted that the effect of the geological formation, which was not considered in low flow scaling in this study, is also important in the response of a watershed base flow to rainfall. Furey and Gupta (2000) showed that unconsolidated geological formations reduce the effectiveness of drainage area for low flow scaling in semiarid and arid regions.

Conclusions

The scaling properties of low flows, represented by annual minimum 7-, 15-, 30-, and 60-day flows, were assessed for the entire region and two subclimatic regions in northern Iran by investigating the log-log relationships between drainage area and PWMs. Across the region, three main conclusions can be drawn. First, the climate heterogeneity of the region may weaken the relationship between drainage area and low flows' probabilistic behavior. In other words, the drainage area is suitable for low flow scaling in regions with homogeneous climate conditions but is not a good predictor for low flow scaling in regions with heterogeneous climate conditions. Second, low flows in the humid region with homogeneous climate condition generally obey a simple scaling law. That means the slope or the scale exponent H is constant and independent of the order of PWMs (k). The mean values of H are 0.6, 0.61, 0.615, and 0.618 for annual minimum mean 7-, 15-, 30-, and 60 days flows, respectively. Third, low flow scaling is not significant for the eastern semiarid region without adding MAR to the regression equation of scaling. It was also found that for the semiarid region, low flows of medium duration are related to the size of drainage area and MAR. This may be due to the effect of the physical relationship of base flow or groundwater and geological formation. The results suggest that other alternative variables such as rainfall or groundwater for low flow scaling in semiarid region should be considered. In general, the effect of climate heterogeneity and the physical properties of the geological formations within the basin should be considered in future low flow regional scaling.

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References

Burlando, P., and Rosso, R. (1996). "Scaling and multiscalling models of depth-duration-frequency curves for storm precipitation." *J. Hydrol.*, 187, 45–64.

Dalrymple, T. (1960). "Flood frequency analysis." *U.S. Geological survey water supply paper 1543-A*, U.S. Geological Survey, Menlo Park, Calif.

Eaton, B., Church, M., and Ham, D. (2002). "Scaling and regionalization of flood flows in British Columbia, Canada." *Hydrolog. Process.*, 16, 3245–3263.

Furey, P. R., and Gupta, V. K. (2000). "Space-time variability of low streamflows in river network." *Water Resour. Res.*, 36, 2679–2690.

Gupta, V. K., Castro, S. L., and Over, T. M. (1996). "On scaling exponents of spatial peak flow from rainfall and river network geometry." *J. Hydrol.*, 187, 81–104.

Gupta, V. K., and Dawdy, D. R. (1995). "Physical interpretations of regional variation in the scaling exponents of flood quantiles." *Hydrolog. Process.*, 9, 347–361.

Gupta, V. K., Mesa, O. J., and Dawdy, D. R. (1994). "Multiscaling theory of flood peaks: regional quantile analysis." *Water Resour. Res.*, 30, 3405–3421.

Gupta, V. K., and Waymire, E. (1990). "Multi scaling properties of spatial rainfall and river flow distributions." *J. Geophys. Res.*, 95(D3), 1999–2009.

Kumar, P., Guttarp, P., and Foufoula-Georgiou, E. (1994). "A probability weighted moment test to assess simple scaling." *Stochastic Hydrol. Hydraul.*, 8, 173–183.

Mantilla, R., Gupta, V. K., and Mesa, O. J. (2006). "Role of coupled flow dynamics and real network structures on Hortonian scaling of peak flows." *J. Hydrol.*, 322, 155–167.

Modarres, R. (2008). "Regional frequency distribution type of low flow in north of Iran by L-moments." *Water Resour. Manage.*, 22, 823–841.

Ogden, F. L., and Dawdy, D. R. (2003). "Peak discharge scaling in small Hortonian watershed." *J. Hydrol. Eng.*, 8(2), 64–73.

Over, T. M., and Gupta, V. K. (1996). "A space-time theory of mesoscale rainfall using random cascade." *J. Geophys. Res.*, 101(D21), 26319–26331.

Pandey, G. R. (1998). "Assessment of scaling behavior of regional floods." *J. Hydrol. Eng.*, 3(3), 169–173.

Schaefer, M. G. (1990). "Regional analysis of precipitation annual maximum in Washington state." *Water Resour. Res.*, 26, 119–131.

Smith, J. A. (1992). "Representation of basin scale in flood peak distribution." *Water Resour. Res.*, 28(11), 2993–2999.

Stedinger, J. R., Vogel, R. M., and Foufoula-Georgiou, E. (1993). "Frequency analysis of extreme events." *Hand book of hydrology*, D. R. Maidment, ed., McGraw-Hill, New York, 18.1–18.66.

Vogel, R. M., and Sankarasubramanian, A. (2000). "Spatial scaling properties of annual streamflow in the United States." *Hydrol. Sci. J.*, 45, 465–476.

Yue, S., and Wang, C. Y. (2004). "Scaling of Canadian low flows." *Stochastic Environ. Res. Risk Assess.*, 18, 291–305.