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Copula-based drought severity-duration-frequency analysis in Iran

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ABSTRACT: Drought is a complex and multi-attribute natural hazard that has worldwide effects. Defined by a commonly used standardized precipitation index (SPI), each drought event is characterized by three correlated attributes: severity, duration and frequency. A probabilistic approach is developed to establish a drought severity-duration-frequency (SDF) relationship. Copulas are employed to construct the joint distribution function of drought severity and duration. Drought frequency, in terms of recurrence interval of drought events, is then related to the copula-based distribution function *via* a conditional distribution function. The derived analytic drought SDF thus becomes a function of univariate distribution functions of drought severity and duration, a copula function which links the fitted univariate models, and the arrival rate of drought events. In this study, rainfall data for the period of 1954–2003 from two gauge stations in Iran, Abadan in the southwestern semi-arid region and Anzali in the north humid region, are employed as an example to illustrate the proposed approach. From the derived drought SDF, drought severity in Anzali station is greater than those in Abadan station for given drought duration and recurrence interval. The results imply that the drought severity in humid region might be more severe if high rainfall fluctuations exist in that region. Copyright © 2009 Royal Meteorological Society

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1. Introduction

Drought is a normal and recurrent phenomenon that affects humid as well as arid regions to some degree. Stemming from an absence of rainfall, severe droughts may lead to inadequate water supplies in urban and agricultural areas. Wilhite (2000) indicated that drought is the costliest natural hazard in the world. Understanding drought characteristics is, therefore, an essential element in well-prepared drought management plans.

Since droughts are dynamic and multi-attributable in nature, simultaneous assessment of the multi-attributes of droughts is important in evaluating the risk of droughts. A significant amount of research (Gupta and Duckstein, 1975; Sen, 1980; Dracup *et al*., 1980; Zelenhastic and Salvai, 1987; Frick *et al*., 1990; Kendall and Dracup, 1992; Loáiciga and Leipnik, 1996; Chung and Salas, 2000; Cancelliere and Salas, 2004) has developed probabilistic methods to investigate properties of droughts. Multiple attributes of droughts have been evaluated in these studies, but significant correlation relationships are not revealed by separate consideration of correlated characteristics. Some researchers including Shiau and Shen (2001), Bonaccorso *et al.* (2003), González and Valdés (2003), Loáiciga (2005) and Kim *et al.* (2006a, 2006b), further extended the univariate analysis to explore bivariate frequency of droughts.

Joint multivariate models of droughts are difficult to establish because different distribution functions are often used to fit various attributes of droughts such as severity and duration. Such difficulty is alleviated by using copulas to link different marginal distributions to construct the joint multivariate distribution function. Copulas are widely used to investigate multivariate distribution in hydrology. However, most research focuses on rainfall and floods (De Michele and Salvadori, 2003; Favre *et al*., 2004; Salvadori and de Michele, 2004a, 2004b; De Michele *et al*., 2005; Shiau *et al*., 2006; Grimaldi and Serinaldi, 2006; Salvadori and de Michele, 2006; Zhang and Singh, 2006; Kao and Govindaraju, 2007; Renard and Lang, 2007; Zhang and Singh, 2007; Evin and Favre, 2008; Kao and Govindaraju, 2008). To our knowledge, Shiau (2006) is the first use the copula method to investigate the drought problem.

Rainfall intensity-duration-frequency (IDF) analysis is a useful multivariate tool in urban drainage design because it relates rainfall intensity, rainfall duration and frequency of occurrence (in terms of recurrence interval) in a single diagram. Dalezios *et al.* (2000) used such a technique to develop drought severity-duration-frequency (SDF) curves in Greece. In practice, rainfall IDF curves or drought SDF curves are derived empirically: no analytic approaches for such multivariate curves have been proposed. In the present study, analytical derivation

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of copula-base, drought SDF curves is developed based on the previous studies of drought analysis using the copula method (Shiau, 2006; Shiau *et al.*, 2007).

The present study aims to investigate multi-attributes of droughts in terms of copula-based drought severity-duration-frequency (SDF) curves. Droughts are defined by the commonly used standardized precipitation index (SPI) (McKee *et al.*, 1993) for 3-month rainfall series. Drought severity and duration are abstracted from observed drought data and fitted by a probabilistic model. Copulas are then used to link the fitted model to construct the joint distribution function and to establish the drought SDF curve. Rainfall records of the period of 1954–2003 for two raingauge stations located at different climatic regions in Iran are selected as an example to illustrate the proposed method and to explore the drought attributes at two different climatic regions.

2. Study Area and Data Used

Iran lies approximately between 45°–65 °E and 25°–45°N and is surrounded by two major mountain ranges, Alborz in the north and Zagros in the west. The highest peak of the Alborz mountain range, with an elevation of 5628 m above mean sea level, is the highest elevation in Iran. These mountains impede Mediterranean moisture systems crossing through the inland region of Iran. Great spatial variation of rainfall, therefore, exists in Iran. The long-term mean annual rainfall in Iran ranges from below 100 mm to exceeding 1000 mm (Modarres, 2006).

In this study, rainfall records from two raingauge stations, Abadan and Anzali (Figure 1), located in different climatic zones are selected to explore their drought characteristics. Abadan is located in southwestern semi-arid Iran and Anzali station is located in northern Iran which is classified as a humid region. The mean annual rainfalls for Abadan and Anzali stations are 155 and 1860 mm, respectively (Table I).

The monthly rainfall records of both stations for the period 1954–2003 are employed in drought analysis. Figure 2 demonstrates the monthly rainfall distribution for both stations. A clear unevenly distributed monthly rainfall pattern is observed. Recently, the standardized precipitation index (SPI), developed by McKee *et al.* (1993), has gained worldwide popularity in drought monitoring and analysis (Patel *et al.*, 2007). The SPI, thus, is used to define droughts in this study. The 3-month SPI is calculated first based on the methodology described by Vicente-Serrano (2006). The 3-month SPI series of the period 1954–2003 for Abadan and Anazli, shown in Figure 3, is used to determine drought characteristics. As drought is defined when the values of SPI fall below zero, a drought event is considered a period with negative SPI values. Drought duration denoted by D, therefore, is the continuous negative-SPI periods, while drought severity denoted by S is the cumulative values of SPI within the drought duration. For convenience, drought severity is taken to be positive, which is defined by:

$$S = -\sum_{i=1}^{D} SPI_i \tag{1}$$

The basic statistics of drought duration and severity for both Abadan and Anzali are also summarized in Table I. Although the number of droughts occurred in the period of 1954–2003 for both stations are nearly the same, both the standard deviation of drought duration and severity for Anazli are larger than those of Abadan. This fact indicates that drought characteristics are highly fluctuating for Anazli.

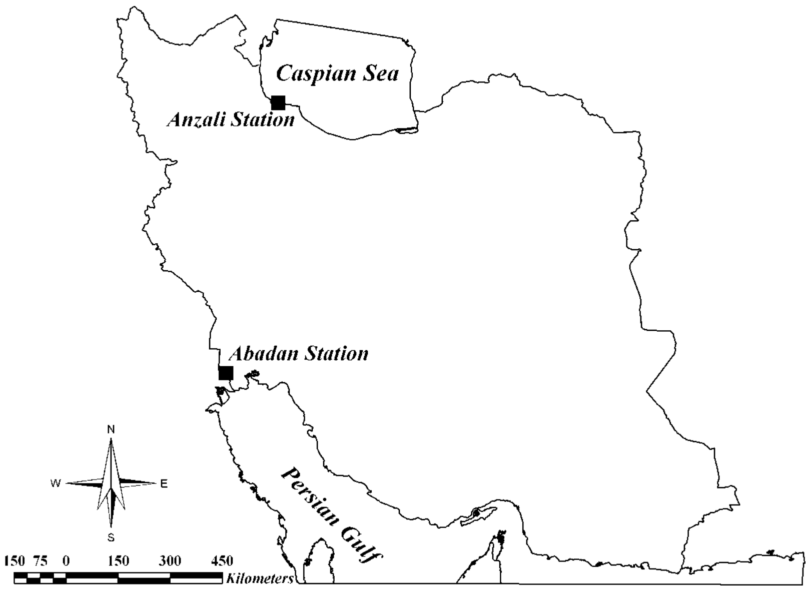

Figure 1. Location map of the two rainfall gauge stations (Abadan and Anzali) in Iran.

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Table I. Basic statistics of rainfall and droughts for Abadan and Anazli.

	Station Climatic zone		Abadan Semi-arid	Anazli Humid
Rainfall	Mean annual rainfall (mm)		155	1860
	Standard deviation (mm)		62	351
	Coefficient of variation (%)		40	18.9
Drought	No. of droughts (1954–2003)		83	82
	Arriving rate (number/year)		1.66	1.64
	Duration (months)	Mean	2.663	3.744
		Standard deviation	2.062	2.698
		Minimum	1	1
		Maximum	9	13
	Severity	Mean	2.061	3.005
		Standard deviation	2.500	2.881
		Minimum	0.03	0.07
		Maximum	13.67	13.59

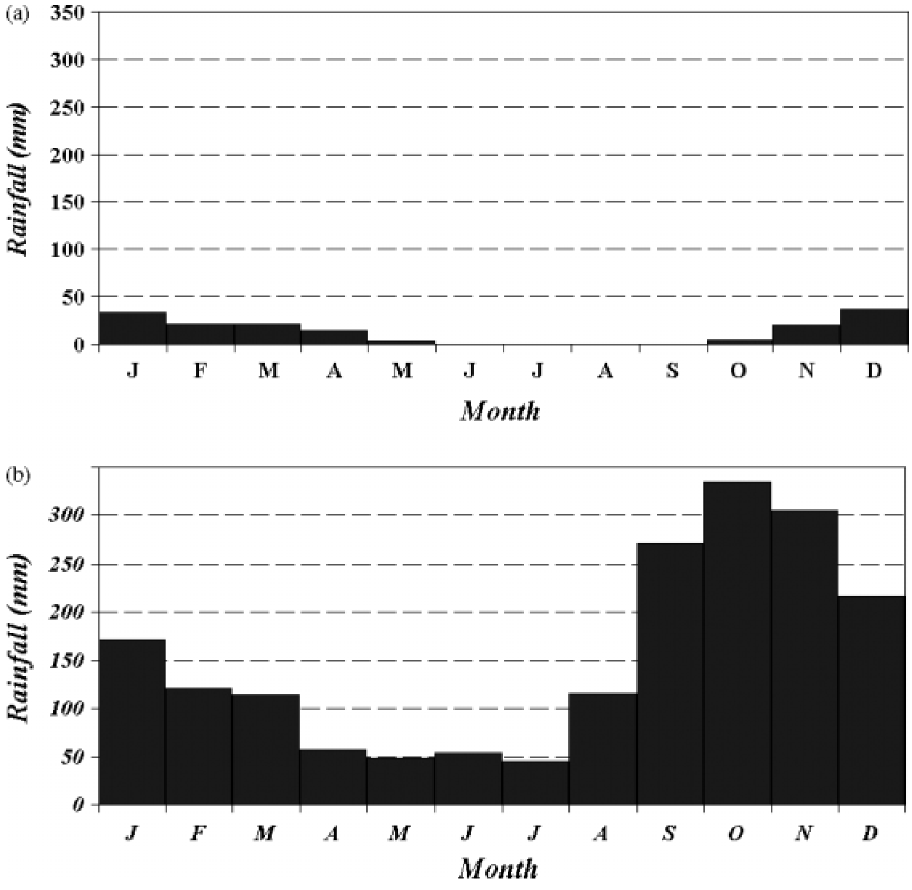

Figure 2. Monthly rainfall distribution for (a) Abadan and (b) Anzali.

3. Methodology

3.1. Definition of copulas

To model a multivariate hydrological process, a joint distribution of such correlated multivariates is an appropriate method to investigate simultaneously relationships among correlated variables. However, multivariate modelling has not been widely used in hydrology since the correlated variables may not be best fitted by the same type of distributions and few models are available for such conditions. Multivariate distribution construction using copulas, developed by Sklar (1959), can overcome such difficulties which arise in practical applications. The essential concept of copulas is that a joint distribution of correlated multivariates can be expressed as a function of univariate marginal distributions. That is, a copula is a function that links univariate marginal distribution functions to construct a multivariate distribution function.

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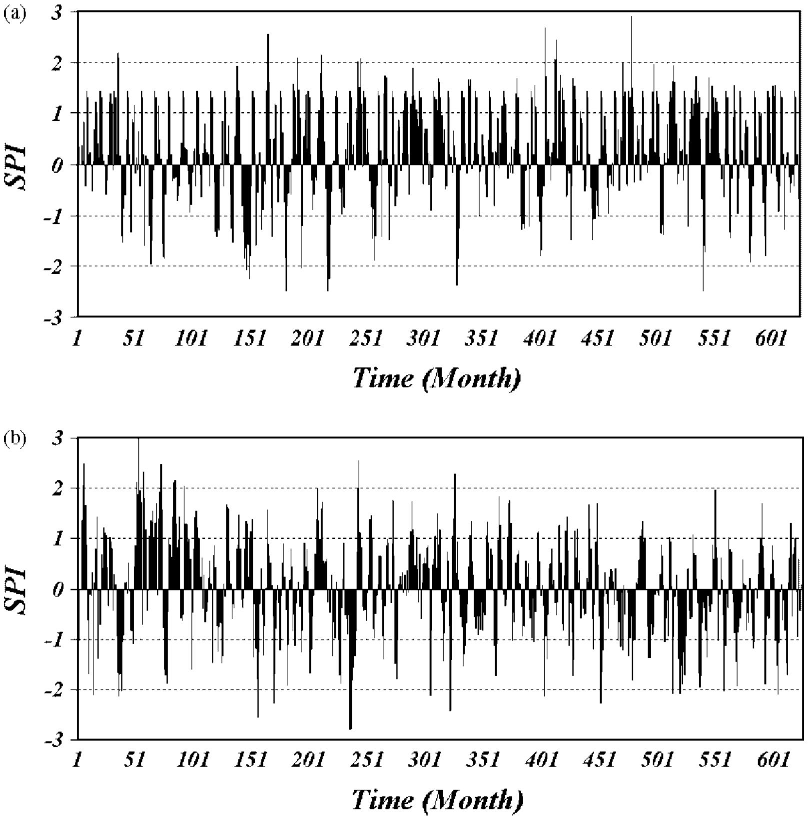

Figure 3. Three-month standardized precipitation index (SPI) series for (a) Abadan and (b) Anzali.

Sklar's Theorem states that if $F_{X_1,\,X_2,\,\ldots,X_m}(x_1, x_2, \ldots, x_m)$ is a multivariate distribution function of m correlated random variables of $X_1,\ X_2,\ \ldots,\ X_m$ with respectively marginal distributions $F_{X_1}(x_1),\ F_{X_2}(x_2),\ \ldots,\ F_{X_m}(x_m)$, then there exists a copula C such that:

$$F_{X_1,\,X_2,\,\ldots,\,X_m}(x_1,\ x_2,\ \ldots,\ x_m) = C(F_{X_1}(x_1),\ F_{X_2}(x_2),\ \ldots,\ F_{X_m}(x_m)) \quad (2)$$

Conversely, for any univariate marginal distributions $F_{X_1}(x_1),\ F_{X_2}(x_2),\ \ldots,\ F_{X_m}\ (x_m)$, and any copula C, the function $F_{X_1,\,X_2,\,\ldots,\,X_m}(x_1,\ x_2,\ \ldots,\ x_m)$ is a joint distribution function with marginal distribution functions $F_{X_1}(x_1),\ F_{X_2}(x_2),\ \ldots,\ F_{X_m}(x_m)$.

Detailed properties and various types of copulas can be founded in Joe (1997), Nelsen (1999), Cherubini *et al.* (2004) and Salvadori *et al.* (2007). Since using copulas offers greater flexibility to select suitable univariate distributions well fitted to the observed data and easier to construct a multivariate distribution, copulas are used in this study to construct the multivariate drought model.

3.2. Copula-based drought severity-duration-frequency relationship

In order to explore the correlated drought characteristics, the drought SDF relationship using copula is proposed in this study. Each drought event is characterized by drought severity and duration and, therefore, is considered a bivariate random variable. The relationship among the drought severity, drought duration and frequency (in terms of recurrence interval) for drought events can be represented by the conditional recurrence interval, which is given by:

$$T_{S|D}(s|d) = \frac{1}{\gamma(1 - F_{S|D}(s|d))} \quad (3)$$

where s and d denote drought severity and drought duration, respectively; $F_{S|D}(s|d)$ is the conditional cumulative distribution function of S given $D = d$, that is $F_{S|D}(s|d) = P(S \le s|D = d)$; $T_{S|D}(s|d)$ is the conditional recurrence interval of S given $D = d$; γ is the arriving rate of the drought events.

The conditional cumulative distribution function $F_{S|D}$ $(s|d)$ relates to the joint cumulative distribution function (JCDF) of drought severity and duration $F_{S,\ D}(s,\ d)$ and the cumulative distribution function (CDF) of drought duration $F_D(d)$ is given by the following relationship:

$$F_{S|D}(s|d) = \frac{\partial F_{S,D}(s,d)}{\partial F_D(d)} \quad (4)$$

where $F_D(d)$ is the CDF of drought duration; $F_{S,\ D}(s,\ d)$ is the JCDF of drought severity and drought duration.

Thus, to establish the theoretical drought SDF relationship, $F_{S,\ D}(s, d)$ and F_D (d) need to be derived first. Since drought severity and duration may not have identical types of distribution functions, the commonly used bivariate distribution functions with same type of

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marginal distributions such as bivariate exponential distribution, are generally not fitable to the observed drought data. Copulas, initialized by Sklar (1959), can model the dependence relationship among random variables independently of their marginal distributions. Therefore, using copulas to construct the JCDF of drought severity and duration can overcome such difficulties if they are fitted by different types of CDFs. The JCDF of drought severity and duration in terms of copulas is a function of univariate CDFs of drought duration and drought severity, which is defined by Equation (2) and given by:

$$F_{S,\ D}(s,\ d) = C(F_S(s),\ F_D(d)) \tag{5}$$

where $F_S(s)$ and $F_D(d)$ are CDFs for drought severity and drought duration, respectively; C is a copula function.

Therefore, the conditional distribution function $F_{S|D}$ $(s|d)$ defined in Equation (4) is also expressed as a function of copula, that is:

$$\begin{aligned} F_{S|D}(s|d) &= \frac{\partial F_{S,D}(s,d)}{\partial F_D(d)} = \frac{\partial C(F_S(s), F_D(d))}{\partial F_D(d)} \\ &= C_{F_S|F_D}(F_S(s)|F_D(d)) \end{aligned} \tag{6}$$

where $C_{F_S|F_D}(F_S(s)|F_D(d))$ is a conditional copula.

The conditional recurrence interval $T_{S|D}(s|d)$ defined in Equation (3), denoting the theoretical drought SDF relationship, thus, becomes:

$$\begin{aligned} T_{S|D}(s|d) &= \frac{1}{\gamma(1 - F_{S|D}(s|d))} \\ &= \frac{1}{\gamma\{1 - C_{F_S|F_D}(F_S(s)|F_D(d))\}} \end{aligned} \tag{7}$$

The derived theoretical drought SDF relationship in terms of copulas, therefore, is a function of univariate CDFs of drought severity $F_S(s)$ and drought duration $F_D(d)$, a copula function, C, which is used to construct the dependence between drought severity and duration, and the arriving rate of drought events γ. This general formula can be applied to any location for any hydrologic variables used to define droughts except that $F_S(s)$, $F_D(d)$, C, and γ need to be fitted from the observed drought data.

4. Results and Discussion

4.1. Univariate CDFs for drought severity and duration

The drought severity and duration for both Abadan and Anzali were calculated from observed droughts defined by SPI, which are demonstrated in Figure 6(a) and (b). Significant correlations observed between drought severity and duration reveals that drought severity and duration should be modelled jointly. In terms of copulas, the joint distribution function is a function of the marginal univariate distribution functions. Thus, the univariate CDFs of drought severity and duration needs to be fitted from the observed drought data first. Generally, drought severity and duration can be fitted to the gamma and exponential distributions, respectively, with probability density functions (PDF) shown below:

$$f_S(s) = \frac{s^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)} e^{-s/\beta}, \quad s > 0 \tag{8}$$

$$f_D(d) = \frac{1}{\lambda} e^{-d/\lambda}, \quad d > 0 \tag{9}$$

where α and β are shape and scale parameters of the gamma distribution and λ is the parameter of the exponential distribution, which need to be estimated from the observed data.

These parameters, drought severity and duration, are estimated by the maximum likelihood method and are summarized in Table II. These fitted models for both stations Abadan and Anazli are shown in Figures 4 and 5, which show good agreement with the observed drought data. The Kolmogorov-Smirnov (K-S) goodness-of-fit test is used to detect whether the proposed models can be used to represent the observed data. The critical values for sample size of 83 (Abadan) and 82 (Anazli) are 0.134 and 0.135, respectively, at the 10% significance level. The maximum deviations between observed data and proposed models of drought duration and severity for Abadan and Anazli are also reported in Table II, which all are less than the critical values. The results indicate that the hypothesis of proposed gamma and exponential distributions to model drought severity and duration for both Abadan and Anazl, respectively, cannot be rejected.

Table II. Parameters of proposed models.

Station		Abadan	$D_{\max}$	Anazli	$D_{\max}$
Duration	λ	2.125	0.077	3.219	0.121
Severity	α	0.737	0.096	0.917	0.066
	β	2.796		3.277	

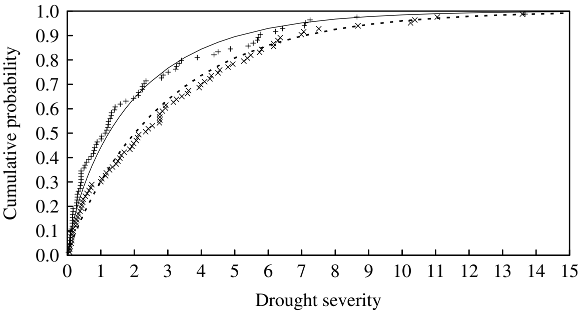

Figure 4. Observed drought severity and fitted gamma model for Abadan and Anzali. Abadan (Observed) +, Abadan (fitted) −, Anzali (Observed) ×, Anzali (fitted) - - - - .

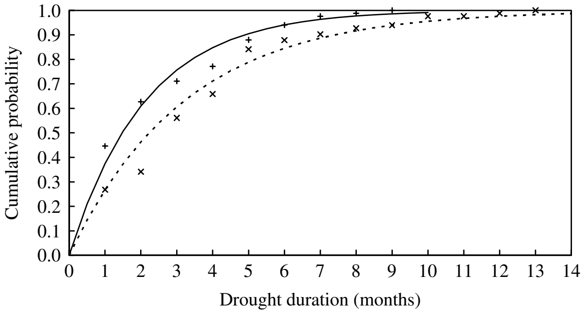

Figure 5. Observed drought duration and fitted exponential model for Abadan and Anzali. Abadan (Observed) +, Abadan (fitted) –, Anzali (Observed) ×, Anzali (fitted) - - - - .

4.2. Copula-based JCDF of drought severity and duration

Since drought severity and duration are modelled by different CDFs, copulas are used to link the fitted models and construct the JCDF of drought severity and duration. In this study, the Clayton copula is employed to model the dependence between drought severity and duration since it is of simple form and is commonly used in hydrology. The copula-based JCDF of drought severity and duration, therefore, become:

$$C(F_S, F_D) = (F_S^{-\theta} + F_D^{-\theta} - 1)^{-\frac{1}{\theta}}, \quad \theta \geq 0 \tag{10}$$

where F_S and F_D are the univariate CDFs for drought severity and duration, respectively; and θ is a parameter used to measure the degree of association between F_S and F_D.

For both the Abadan and Anzali stations considered in this study, drought severity and duration are modelled separately by gamma and exponential distributions, which are described in Equations (8) and (9) with parameters summarized in Table II. The parameter of the Clayton copula is estimated by the method of inference function for margins (IFM) (Joe, 1997). The values of these parameters for Abadan and Anzali are 1.527 and 1.497, respectively. Therefore, the copula-based JCDF of drought severity and duration for Abadan is:

$$F_{S,D}(s,d) = \left(F_S(s)^{-1.527} + F_D(d)^{-1.527} - 1\right)^{-\frac{1}{1.527}} \tag{11}$$

where F_S and F_D are CDFs for drought severity and duration, respectively. With parameters summarized in Table II, F_S and F_D become:

$$F_S(s) = \int_0^s \frac{t^{-0.263}}{2.796^{0.737}\Gamma(0.737)} \mathrm{e}^{-t/2.796} \mathrm{d}t \tag{12}$$

$$F_D(d) = 1 - \mathrm{e}^{-d/2.125} \tag{13}$$

Similarly, the copula-based JCDF associated with univariate CDFs of drought severity and duration for Anazli are:

$$F_{S,D}(s,d) = (F_S(s)^{-1.497} + F_D(d)^{-1.497} - 1)^{-\frac{1}{1.497}} \tag{14}$$

$$F_S(s) = \int_0^s \frac{t^{-0.083}}{3.277^{0.917}\Gamma(0.917)} \mathrm{e}^{-t/3.277} \mathrm{d}t \tag{15}$$

$$F_D(d) = 1 - \mathrm{e}^{-d/3.219} \tag{16}$$

The contours of joint probability associated with the observed drought data for Abadan and Anzali are demonstrated in Figure 6(a) and (b), respectively.

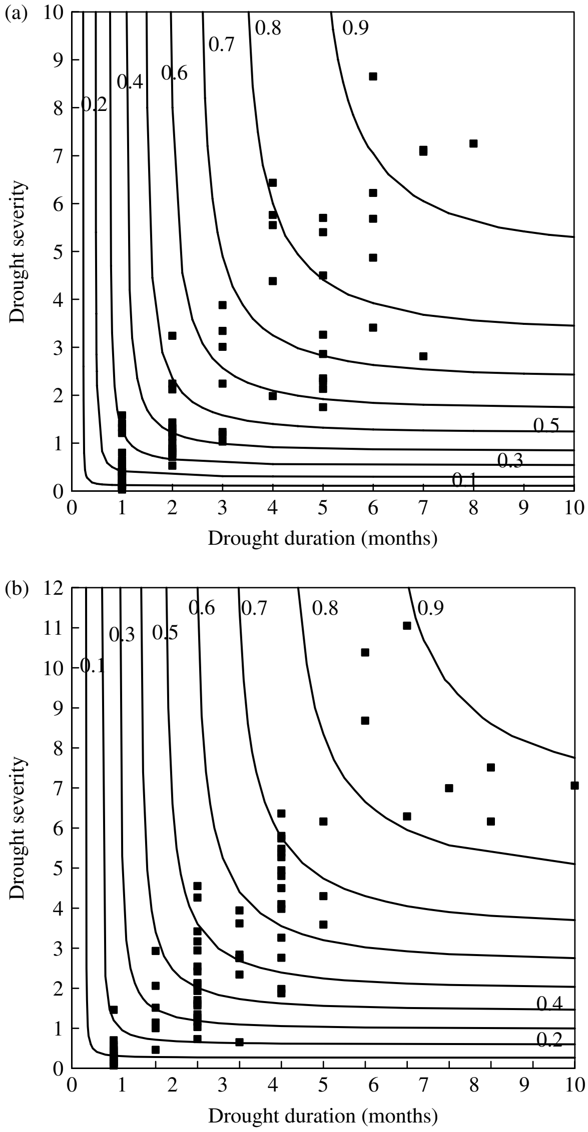

Figure 6. The contours of joint probabilities of drought severity and duration for (a) Abadan and (b) Anzali.

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4.3. Copula-based drought SDF curves

The conditional Clayton copula in terms of $F_S(s)$ and $F_D(d)$ has the following form (Joe, 1997):

$$C_{F_S|F_D}(F_S(s)|F_D(d)) = \left\{1 + F_D(d)^{\theta}(F_S(s)^{-\theta} - 1)\right\}^{-\frac{1}{\theta}-1} \quad (17)$$

Since the conditional distribution function $F_{S|D}(s|d)$ defined in Equation (4) has the identical form with conditional copula, the copula-based drought SDF curve is therefore given by:

$$T_{S|D}(s|d) = \frac{1}{\gamma\left[1 - \left\{1 + F_D\ (d)^{\theta}\left(F_S\ (s)^{-\theta} - 1\right)\right\}^{-\frac{1}{\theta}-1}\right]} \quad (18)$$

The arrival rate of drought events for Abadan and Anazli are 1.66 and 1.64, respectively (Table II). Thus, the theoretical drought SDF relationships for both Abadan and Anazli become:

$$T_{S|D}\ (s|d) = \frac{1}{1.66\left[1 - \left\{1 + F_D\ (d)^{1.527}\left(F_S\ (s)^{-1.527} - 1\right)\right\}^{-\frac{1}{1.527}-1}\right]} \quad (19)$$

$$T_{S|D}\ (s|d) = \frac{1}{1.64\left[1 - \left\{1 + F_D\ (d)^{1.497}\left(F_S\ (s)^{-1.497} - 1\right)\right\}^{-\frac{1}{1.497}-1}\right]} \quad (20)$$

The developed drought SDF curves for Abadan and Anazli with selected recurrence intervals of 1, 2, 5, 10 and 20 years are illustrated in Figure 7(a) and (b), respectively. These SDF curves show a concave down pattern. i.e. these curves increase with drought duration, but at a decreasing rate(that is, drought severity increases rapidly during the stage of short drought duration). As drought duration continuously increases, drought severity increases mildly and then approximately approaches a constant value. The constant value of drought severity for longer drought duration and a given recurrence interval can be derived analytically from Equation (17). Since the value of $F_D(d)$ is approximately equal to one for very large d, the constant value of drought severity for given recurrence interval is thus determined by:

$$s = F_S^{-1}\left\{\left(1 - \frac{1}{\gamma T}\right)^{\frac{1}{1+\theta}}\right\} \quad (21)$$

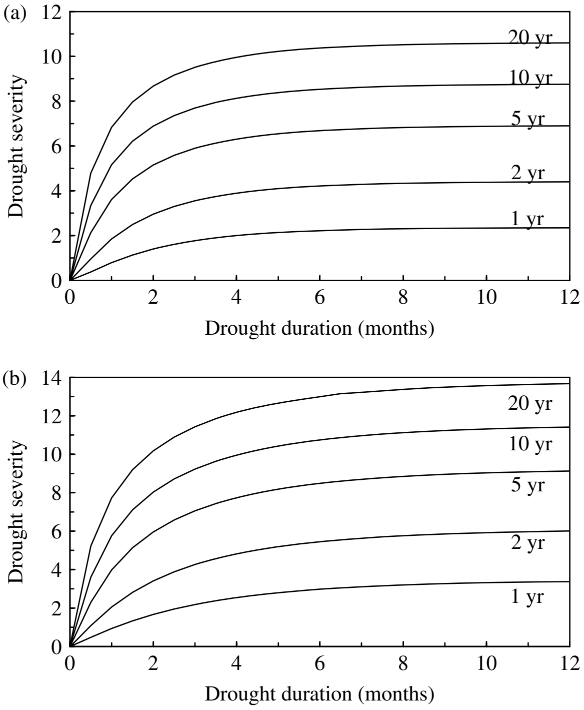

Figure 7. The drought SDF curves for (a) Abadan and (b) Anzali.

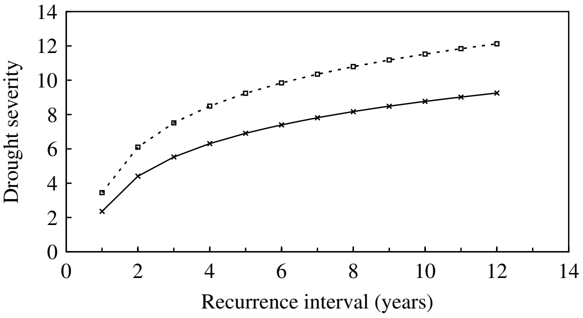

Figure 8. The drought severity limit of longer drought duration for (a) Abadan and (b) Anzali. Abadan ×, Anzali □.

For the models and parameters estimated from the observed drought data, the constant values of drought severity of longer drought duration shown in Figure 7 for Abadan and Anzali with various recurrence intervals are demonstrated in Figure 8. Generally, drought severity at Anzali is greater than the value at Abadan for the same duration and recurrence interval. The more severe droughts occurred in the humid region is attributed to the highly fluctuating rainfall there.

5. Summary and Conclusions

As drought is a critical component in water resources planning and management, understanding drought characteristics is an essential step to construct efficient mitigation measures for drought-related problems. In this study, drought is defined by the standardized precipitation index (SPI) series when the values below zero. Each drought event is characterized by drought ‘severity and duration', which are separately modelled by a probabilistic distribution function. A copula function is then employed to link the fitted model and to construct joint distribution function of drought severity and duration. Based on the conditional distribution function and the derived copula-based joint distribution function, the analytical expression of drought severity-duration-frequency (SDF) are then derived. The copula-based drought SDF curves become a function of univariate distribution function of drought severity and duration, a copula function which is used to describe the dependence structure of drought severity and duration, and the arriving rate of drought event.

The drought SDF curves of various recurrence intervals for two rainfall gauge stations at different climatic regions in Iran are developed by the proposed approach. These drought SDFs are concave down curves. That is, drought severity increases with drought duration, but at a decreasing rate. As drought duration continuously increases, drought severity initially increase rapidly, then mildly, finally approaching a constant value. The constant value of each SDF curve can also be theoretically derived because that the value of probability approximately approaches one for longer drought durations. The merit of analytical expression of the drought SDF curves can be seen here. According to the derived drought SDF curves, drought severity at Anzali is generally greater than the value at Abadan for the same duration and recurrence interval. The more severe droughts occurred in the humid region is attributed to the fact that highly fluctuating rainfall exists in that region.

References

Bonaccorso B, Cancelliere A, Rossi G. 2003. An analytical formulation of return period of drought severity. *Stochastic Environmental Research and Risk Assessment* **17**: 157–174.

Cancelliere A, Salas JD. 2004. Drought length properties for periodic-stochastic hydrologic data. *Water Resources Research* **40**: W02503, DOI: 10.1029/2002WR001750.

Cherubini U, Luciano E, Vecchiatio W. 2004. *Copula Methods in Finance*. John Wiley & Sons: Chichester.

Chung CH, Salas JD. 2000. Drought occurrence probabilities and risks of dependent hydrologic processes. *Journal of Hydrologic Engineering* **5**: 259–268.

Dalezios NR, Loukas A, Vasiliades L, Liakopoulos E. 2000. Severity-duration-frequency of droughts and wet periods in Greece. *Hydrological Sciences Journal* **45**: 751–769.

De Michele C, Salvadori G. 2003. A generalized Pareto intensity-duration model of storm rainfall exploiting 2-copulas. *Journal of Geophysical Research* **108**: 4067, DOI:10.1029/2002JD002534.

De Michele C, Salvadori G, Canossi M, Petaccia A, Rosso R. 2005. Bivariate statistical approach to check adequacy of dam spillway. *Journal of Hydrologic Engineering* **10**: 50–57.

Dracup JA, Lee KS, Paulson EG. 1980. On the statistical characteristics of drought events. *Water Resources Research* **16**: 289–296.

Evin G, Favre AC. 2008. A new rainfall model based on the Neyman-Scott process using cubic copulas. *Water Resources Research* **44**: W03433, DOI: 10.1029/2007WR006054.

Favre AC, El Adlouni S, Perreault L, Thiemonge N, Bobee B. 2004. Multivariate hydrological frequency analysis using copulas. *Water Resources Research* **40**: W01101, DOI: 10.1029/2003WR002456.

Frick DM, Bode D, Salas JD. 1990. Effect of drought on urban water supplies. I: drought analysis. *Journal of Hydraulic Engineering* **116**: 733–753.

González J, Valdés JB. 2003. Bivariate drought recurrence analysis using tree ring reconstructions. *Journal of Hydrologic Engineering* **8**: 247–258.

Grimaldi S, Serinaldi F. 2006. Asymmetric copula in multivariate flood frequency analysis. *Advances in Water Resources* **29**: 1155–1167.

Gupta VK, Duckstein L. 1975. A stochastic analysis of extreme droughts. *Water Resources Research* **11**: 221–228.

Joe H. 1997. *Multivariate Models and Dependence Concepts*. Chapman and Hall: New York.

Kao SC, Govindaraju RS. 2007. A bivariate frequency analysis of extreme rainfall with implications for design. *Journal of Geophysical Research* **112**: D13119, DOI: 10.1029/2007JD008522.

Kao SC, Govindaraju RS. 2008. Trivariate statistical analysis of extreme rainfall events via the Plackett family of copulas. *Water Resources Research* **44**: W02415, DOI: 10.1029/2007WR006261.

Kendall DR, Dracup JA. 1992. On the generation of drought events using an alternating renewal-reward model. *Stochastic Hydrology and Hydraulics* **6**: 55–68.

Kim TW, Valdes JB, Aparicio J. 2006a. Spatial characterization of droughts in the Conchos River Basin based on bivariate frequency analysis. *Water International* **31**: 50–58.

Kim TW, Valdes JB, Yoo C. 2006b. Nonparametric approach for bivariate drought characterization using Palmer drought index. *Journal of Hydrologic Engineering* **11**: 134–143.

Loáiciga HA, Leipnik RB. 1996. Stochastic renewal model of low-flow streamflow sequences. *Stochastic Hydrology and Hydraulics* **10**: 65–85.

Loáiciga HA. 2005. On the probability of droughts: the compound renewal model. *Water Resources Research* **41**: W01009, DOI:10.1029/2004WR003075.

McKee TB, Doesken J, Kleist J. AMS. 1993. The relationship of drought frequency and duration to time scales. *Proceedings of the 8th Conference on Applied Climatology.*, Boston; 179–184.

Modarres R. 2006. Regional precipitation climates of Iran. *New Zealand Journal of Hydrology* **45**: 13–27.

Nelsen RB. 1999. *An Introduction to Copulas*. Springer-Verlag: New York.

Patel NR, Chopra P, Dadhwal VK. 2007. Analyzing spatial patterns of meteorological drought using standardized precipitation index. *Meteorological Applications* **14**: 329–336.

Renard B, Lang M. 2007. Use of a Gaussian copula for multivariate extreme value analysis: some case studies in hydrology. *Advances in Water Resources* **30**: 897–912.

Salvadori G, de Michele C. 2004a. Analytical calculation of storm volume statistics involving Pareto-like intensity-duration marginals. *Geophysical Research Letters* **31**: L04502. DOI:10.1029/2003 GL018767.

Salvadori G, de Michele C. 2004b. Frequency analysis via copula: theoretical aspects and applications to hydrological events. *Water Resources Research* **40**: W12511, DOI:10.1029/2004WR003133.

Salvadori G, De Michele C. 2006. Statistical characterization of temporal structure of storms. *Advances in Water Resources* **29**: 827–842.

Salvadori G, De Michele C, Kottegoda NT, Rosso R. 2007. *Extremes in Nature: An Approach Using Copulas*. Springer: Dordrecht.

Sen Z. 1980. Statistical analysis of hydrologic critical droughts. *Journal of Hydraulics Division* **106**: 99–115.

Shiau JT, Shen HW. 2001. Recurrence analysis of hydrologic droughts of differing severity. *Journal of Water Resources Planning and Management* **127**: 30–40.

Shiau JT. 2006. Fitting drought duration and severity with two-dimensional copulas. *Water Resources Management* **20**: 795–815.

Shiau JT, Wang HY, Tsai CT. 2006. Bivariate frequency analysis of floods using copulas. *Journal of the American Water Resources Association* **42**: 1549–1564.

Shiau JT, Feng S, Nadarajah S. 2007. Assessment of hydrological droughts for the Yellow River, China using copulas. *Hydrological Processes* **21**: 2157–2163.

Sklar K. 1959. *Fonctions de repartition à n dimensions et leura marges.* Publications de l'Institut de Statistique de l'Université: Paris, 8, 229–231.

Vicente-Serrano SM. 2006. Differences in spatial patterns of drought on different time scales: an analysis of the Iberian Peninsula. *Water Resources Management* **20**: 37–60, DOI: 10.1007/s11269-006-2974-8.

Wilhite DA. 2000. Drought as a natural hazard: concepts and definitions. In: *Drought: A Global Assessment*, Wilhite DA (ed.). Routledge: New York.

Zelenhastic E, Salvai A. 1987. A method of streamflow drought analysis. *Water Resources Research* **23**: 156–168.

Zhang L, Singh VP. 2006. Bivariate flood frequency analysis using the copula method. *Journal of Hydrologic Engineering* **11**: 150–164.

Zhang L, Singh VP. 2007. Bivariate rainfall frequency distributions using Archimedean copulas. *Journal of Hydrology* **332**: 93–109.