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Regional Frequency Distribution Type of Low Flow in North of Iran by L-moments

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Abstract A regional low flow frequency analysis in the north of Iran using L-moments was carried out. Low flow events have been represented by the 7-day annual minimum series and the L-moments approach was used to assign these data into homogenous regions. According to the homogeneity measure and climatic properties, two subdivisions were found – one in the west of the study area having a homogenous assemblage of sites, and one in the east in which the sites were found to be heterogeneous. The regional low flow frequency distribution was derived for the western division using L-moments and goodness of fit tests were used to evaluate which of a number of possible distributions best represented this. The evaluation suggested that the Generalized Logistic distribution gives the best overall result. For heterogeneous subdivision, the performance of the 2-parameter distribution such as 2-parameter Log Normal, Normal and Gamma distributions gave the best result for the majority of sites. Regional and at-site frequency curves were also compared for the western division, which showed that the quantile estimates could be very different in the upper and lower tails of the distribution. The influence on flow regime and watershed properties on the type of the best fit distribution was investigated which showed that the 2 and 3 parameter distributions do not have a clear relationship with climatic and physiographic characteristics of the watersheds except the watershed area. This may result in simple scaling laws of low flows.

Keywords Low flow · L-moments · Homogeneity · GLOG distribution

1 Introduction

Estimation of low flow statistics is crucial for various hydrologic studies such as water quality management, determination of minimum downstream flow requirement for hydropower, cooling, irrigation system design and assessing the impact of prolonged droughts on aquatic ecosystems.

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At site estimation of low flow statistics at gauged watersheds is not a difficult task for hydrologists. However, Low-flow regionalization, which is fitting frequency distribution and estimating low flow statistics to non-gauged watersheds, has been a problem with which hydrologists have always concerned.

Many investigators have attempted to develop regional hydrologic models to estimate Low-flow statistics at non-gauged watersheds. Most of them have been trying to relate various geomorphic, geologic, climatic and topographic characteristics of the watersheds to low flow statistics (i.e. Tasker 1972, Dingman and Lawlor 1995, Vogel and Kroll 1990, among others). This includes the use of index flood method, multiple regression techniques and other traditional methods of regionalization used to delineate a homogenous region. The resulting regions are sometimes not hydrologically homogenous, especially if the spatial variability of the physiographic or hydrologic characteristics is large. In addition, these methods may produce substantial errors in the estimation of quantiles for the basins close to the regional boundaries.

Greenwood et al. (1979) introduced the concept of probability weighted moments (PWMs) and Hosking (1986, 1990) defined L-moments as linear combinations of PWMs (Rao and Hamed 1997). Hosking and Wallis (1993) extended the use of L-moments and developed useful statistics for regional frequency analysis which measure discordancy, regional homogeneity and goodness of fit. Regional flood frequency analysis has typically employed these methods. Parida et al. (1998) applied regional flood frequency analysis in India using L-moments and Index flood procedure and found that 3-parameter log-normal distribution (LN3) is an appropriate distribution for the region. Kumar et al. (2003) carried out a regional frequency analysis based on L-moments and concluded that generalized extreme value (GEV) distribution is a robust distribution for their region in India. Lim and Lye (2003) found that generalized extreme value and generalized logistic distributions were appropriate for the distribution of extreme flood events in the Sarawak region of Malaysia. However, few studies have been conducted on low flow frequency analysis (ARIDE 1999; Kroll and Vogel 2002). Durrans and Tomic (1996) applied the methods of regionalization to estimate low flows in 128 gauged stations in the USA and concluded that the log-pearson 3 distribution (LPIII) is a suitable candidate for low flow modeling. Pearson (1995) analyzed minimum low flow for 500 catchments in New Zealand. Kroll and Vogel (2002) used the L-moments to identify the probability of low flows in the USA and recommended Pearson III (PIII) and LN3 for the USA. More recently, Chen et al. (2006) carried out a regional low flow frequency analysis for the south of China and recommend the LN3 distribution function for the region.

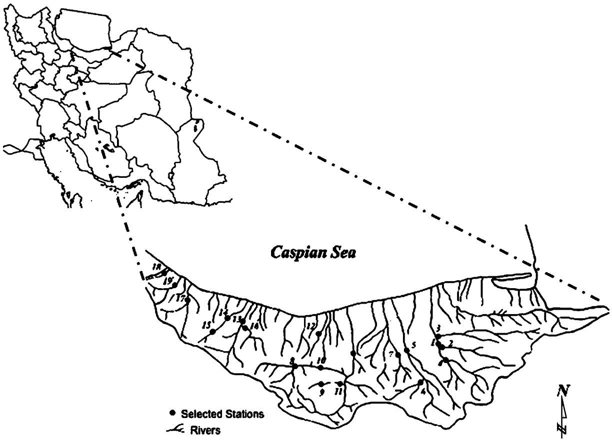
2 Methodology

2.1 Study Area

The present study investigates regional low flow frequency analysis in Mazandaran province in the north of Iran using L-moments. The location map of the study area is shown in Fig. 1. The mean annual rainfall (MAR) in the region ranges from more than 1,000 mm in the west to 300 mm in the east of Mazandaran Province. Most of the eastern part of the province is located at an elevation of less than 500 m above sea level, and is classified as a semi arid region.

The western humid part has also low elevation but the high amount of precipitation makes it more suitable for agricultural activities. Because of these different climatic and physiographic features, the region is first divided into three parts, as required by the

Fig. 1 Location of Study area, Mazandaran Province, North of Iran (from Zareiy 1999, Scale: 1:2,500,000)



homogeneity test in Section 4, based on MAR trend from far west to east (Fig. 2). Mazandran province is also an important region for agricultural production which directly depends on river water resources. Low flow frequency analysis will provide essential information regarding the risks of industrial development and water quality management during times of low flow, i.e. summer season, such as water pollution by pesticides and other industrial waste constituents. Such pollutants can also be harmful to fisheries in Caspian Sea and agricultural activities, which are the main source of rice production in the country. In spite of this, a sufficient streamflow measurement network does not exist.

Neither is there a comprehensive low flow study in this region except for that conducted by Zareiy (1999), who tried to regionalize at site frequency analysis to non-gauged watersheds based on multiple regression nor the method of L-moments method has been applied to Iran yet. The data set contains annual minimum 7-day flow, without any occurrence of zero flow, as a standard streamflow drought index of 19 stations in the region (Table 1). All rivers in the region originate in the Zagros Mountains in the south of the study area. Summer rainfall is also important in the region, which produces no zero value for 7-day low flow with 10-year return period (7dQ10) flow even in dry season.

2.2 Review of L-moment Method

Probability weighted moments were defined by Greenwood et al. (1979) as follows:

$$M_{p,r,s} = \int_{\circ}^{1} [(F)]^{p} F^{r}(1-F)^{2} dF \tag{1}$$

The following two moments are usually considered (Rao and Hamed 1997):

$$M_{1,\circ,s} = \alpha_s = \int_{\circ}^{1} x(F)(1-F)^{s} dF \tag{2}$$

$$M_{1,r,\circ} = \beta_r = \int_{\circ}^{1} x(F)\, F^{r} dF \tag{3}$$

α_s and β_r are related as follows:

$$\alpha_s = \sum_{k=0}^{s} \binom{s}{k} (-1)^{k} \beta_k \tag{4}$$

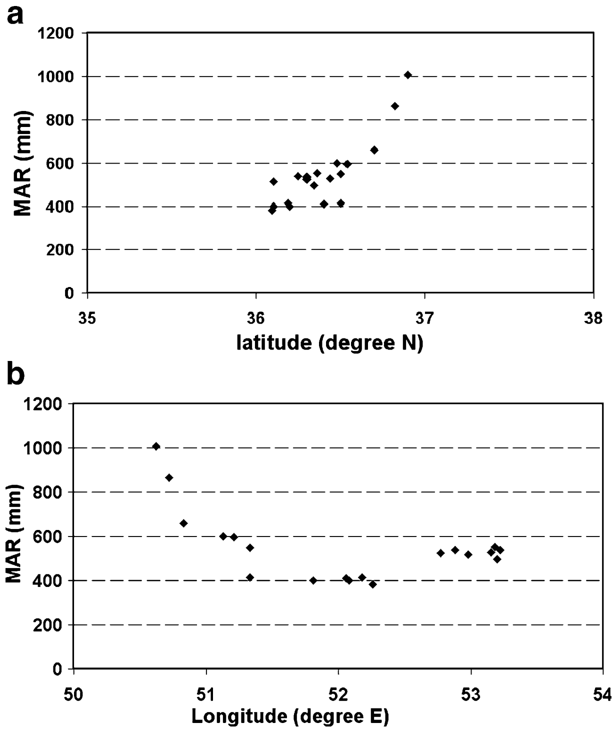


Fig. 2 Mean Annual Rainfall (MAR) variation against **a** latitude and **b** longitude of stations

$$\beta_r = \sum_{k=0}^{r} \binom{r}{k} (-1)^k \alpha_k \tag{5}$$

Hosking (1990) defined the L-moments λ_{r+1} in terms of α_s and β_r

$$\lambda_{r+1} = (-1)^r \sum_{k=0}^{r} p^*_{r,k} \alpha_k = \sum_{k=0}^{r} p^*_{r,k} \beta_k \tag{6}$$

where

$$p^*_{r,k} = (-1)^{r-k} \binom{r}{k} \binom{r+k}{k} \tag{7}$$

The unbiased PWM samples can then be calculated from the following equations:

$$a_s = \frac{1}{n} \frac{\sum_{i=1}^{n} \binom{n-i}{s} x_i}{\binom{n-1}{s}} \tag{8}$$

$$b_s = \frac{1}{n} \frac{\sum_{i=1}^{n} \binom{i-1}{r} x_i}{\binom{n-1}{r}} \tag{9}$$

Table 1 Watershed properties and homogeneity and discordancy measures for selected stations in the region

Station numbers	Name	Sample size	Elevation (m)	Area (km^2)	Mean annual rainfall	Run test's statistic	Run test's *p*-value	LCv	LCs	LCk	D
1	Rig Chesmeh	43	100	2,709	553	0.02	0.96	0.18	0.09	0.16	0.25
2	Varan	16	80	1,191	496	0.02	0.96	0.28	0.18	0.07	1.06
3	Gamrud	10	140	884	529	0.01	0.96	0.48	0.28	0.06	2.88
4	Soleymantangeh	38	1,380	1,256	539	−1.15	0.25	0.18	0.04	0.15	0.20
5	Shirgah	42	975	1,772	537	−1.09	0.27	0.26	0.12	0.11	0.18
6	Alahband	10	375	518	516	0.34	0.73	0.24	0.06	0.06	0.84
7	Gharantalar	43	350	403	525	−0.31	0.76	0.42	0.22	0.14	1.8
8	Baladeh	13	330	750	400	−1.14	0.25	0.22	−0.05	0.14	0.36
9	Namarestagh	10	2,080	119	400	0.03	0.94	0.23	0.19	0.24	0.66
10	Razan	23	860	1,204	414	0.02	0.96	0.21	0.01	0.08	0.66
11	Panjab	17	1,360	253	383	0.01	0.98	0.10	0.09	0.32	2.05
12	Tangeh Lavij	34	1,240	104	410	−1.26	0.18	0.23	0.09	0.23	0.60
13	Polzoghal	41	102	1,544	550	−1.9	0.1	0.17	0.11	0.17	0.34
14	Valt	14	650	328	597	−0.77	0.44	0.18	0.0005	0.13	0.46
15	Kalardasht	35	220	197	601	0.74	0.46	0.20	0.26	0.25	1.39
16	Doab	15	400	627	413	−1.06	0.29	0.21	0.13	0.020	0.18
17	Haratabar	17	280	768	660	−1.00	0.32	0.27	−0.15	0.17	3.99*
18	Ramsar	20	270	136	1,006	0.04	0.93	0.22	0.006	0.07	0.77
19	Ganksar	19	175	415	865	−0.49	0.62	0.28	0.17	0.18	0.31

*Shows discordant site

Sample L-moments are calculated by substituting sample estimates of a_s and b_s in the place of α_s and β_r in Eq. 6. As an alternative, plotting position estimators of sample PWMs are obtained from the following equations:

$$a_s = \widehat{\alpha}_s = \frac{1}{n}\sum_{i=1}^{n}(1 - P_{i:n})^s x_i \tag{10}$$

$$b_r = \widehat{\beta}_r = \frac{1}{n}\sum_{i=1}^{n} P_{i:n}{}^r x_i \tag{11}$$

where $P_{i:n}$=plotting position. Plotting position is a tool for visual inspection and can be used to compare sample frequency distribution with population frequency distribution. Rao and Hamed (1997) suggest that the relationship defined by $P_{i:n} = (1 - 0.35)/n$ gives better estimates of the parameters for both Generalized Extreme Value (GEV) (Hosking et al. 1985) and Generalized Pareto (Hosking and Wallis 1987). L-moment ratios are then defined by Hosking (1986, 1990) in Eqs. 12 and 13:

$$\tau = \lambda_1/\lambda_2 \tag{12}$$

$$\tau_r = \lambda_r/\lambda_2 \quad r \geq 3 \tag{13}$$

where λ_1 is measure of the location, τ is measure of scale and dispersion (L-Coefficient of Variation, LCv), τ_3 is measure of skewness (L-Coefficient of skewness, LCs) and τ_4 is measure of kurtosis (L-Coefficient of kurtosis, LCk). The moment ratio diagram (MRD) is an easy way to identify regional homogeneity of a region. Rao and Hamed (1997) used MRD to identify homogenous regions within the Wabash River basin for flood frequency analysis. Kroll and Vogel (2002) used sample estimates of LCv versus LCs for 7-day lowflows in the United States. L-moment diagrams allow visual comparison of sample estimates to population values of L moments (Stedinger et al. 1993) and are always preferred to product moment ratio diagrams for goodness-of-fit tests (Vogel and Fennessey 1993; ARIDE 1999).

3 Results and Discussions

3.1 Test for Randomness and Homogeneity

The preliminary process of data (annual minimum flow) is achieved by carrying out the test for randomness and homogeneity. The homogeneity of low flow time-series was determined by Thom's test (or run test) at the 95% confidence level. This test assesses the variations of the time-series from a central value, usually the median. Thom's test is described in detail by Rodrigo et al. (1999). The run test's statistics for 7-day low flow time series of selected stations are given in Table 1. It is clear that all low flow time series are homogeneous because $p>0.05$.

Autocorrelation functions (ACF) are commonly used for checking randomness in a data set. This randomness is ascertained by computing autocorrelations for data values at varying time lags. If random, such autocorrelations should be near zero for any and all time-lag separations. If non-random, then one or more of the autocorrelations will be significantly non-zero. These functions were used to test the randomness of low flow time-series and all of the ACFs show the randomness of low flow time series. For example, the autocorrelation functions of two stations, Rigcheshmeh and Polzighal, are presented in Fig. 3. None of the coefficients crosses the line of 95% significant level and thus, it is clear that the low flow time series are time independent.

3.2 L-moment Diagrams

Figures 4 and 5 show MRDs for all stations in the region. A high degree of heterogeneity can be identified from the L-moment ratio diagrams (MRDs). Data points are widely scattered in Figs. 4 and 5. Two statistics introduced by Hosking and Wallis (1993) as alternative methods to identify regional homogeneity are discussed in the following section.

Theoretical relationships among L-Cv and LCs in Fig. 4 were constructed using polynomial approximations developed by Vogel and Wilson (1996). Figure 5 was constructed using the FREQ program developed by Rao and Hamed (2000) for flood regional frequency analysis.

3.3 Homogeneity and Discordancy Test

Hosking and Wallis (1993) derived two statistics to test the homogeneity of a region. The first statistic is used as a discordancy measure. A site is considered to be unusual if the discordancy measure (D) is larger than 3. Other statistics applied for homogeneity test are three heterogeneity measures (H_i), namely, H_1, H_2 and H_3 with respect to LCv scatter, LCv–LCs and LCv–LCv, respectively. A region is homogenous if any of Hi is less than 1, possibly heterogeneous if Hi is between 1 and 2, and definitely heterogeneous if Hi is

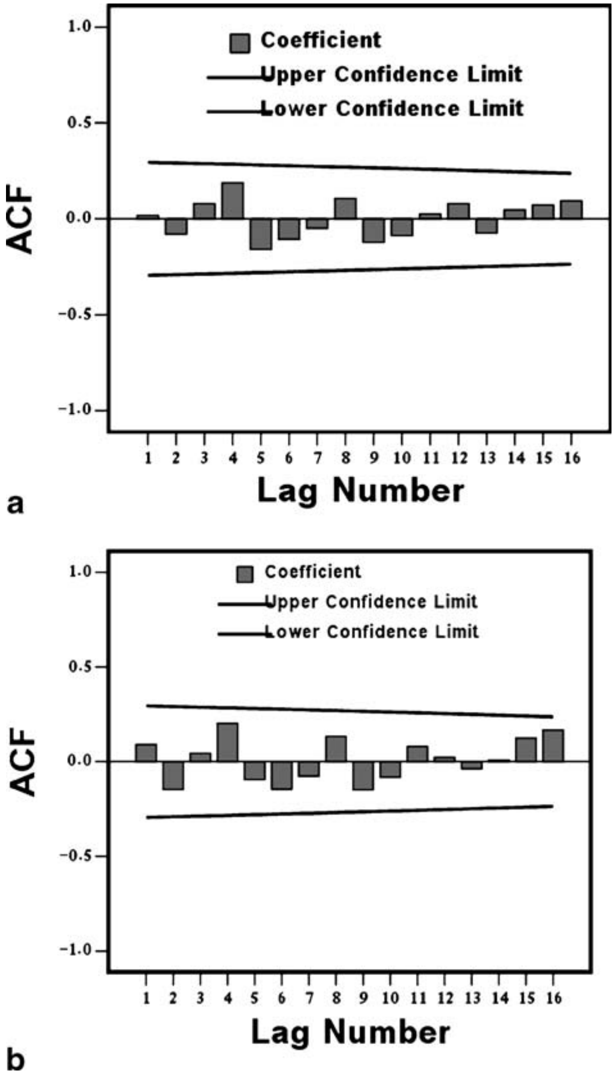
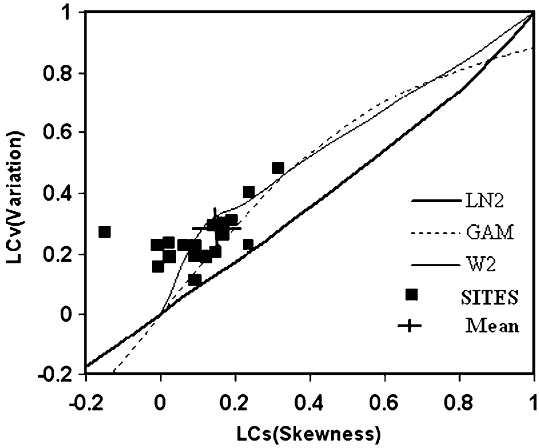


Fig. 3 Autocorrelation function of 7-day low flow time series for Rigchesmeh **a** and Polzoghal **b** stations

greater than 2 (Hosking and Wallis 1993). The results of discordancy measurements are presented in Table 1. In this table, there is only one station with (D) greater than 3. The results of homogeneity measures are also presented in this table. If we remove the discordant station from the list and recalculate the homogeneity, the region remains heterogeneous (H_1=7.22, H_2=0.29, H_3=−1.44). In summery, all relevant statistics indicate that the region should be considered heterogeneous.

Consequently, we subdivided the region into three parts: west, middle and east, based on the decreasing trend of rainfall from west to east. Homogeneity measures were calculated for each subdivision using the FORTRAN computer program developed by Hosking (1991) are presented in Table 2. Because Hosking and Wallis (1993) suggested that H_1 is enough to indicate regional homogeneity, we referred to H_1 to accept or reject homogeneity. Their suggestion is most likely drawn from the fact that H_1 is calculated based on the coefficient

Fig. 4 LC_V–LCs moment ratio diagram for 19 stations in North of Iran



of variation, and for realistic hydrologic regions, it has been illustrated that the coefficient of variation has a high value of regionalization (Stedinger and Lu 1995). According to the result of the homogeneity test, the west region is homogenous while the two other subdivisions are heterogeneous.

To obtain a larger homogeneous region, we subdivided the region again into two subdivisions namely "All West" and "All East" according to rainfall gradient of the province, Fig. 2b, and recalculated the homogeneity statistics. Table 2 shows the results of the process of measuring homogeneity. These results are presented after removing the discordant station.

Figures 6 and 7 show LCV–LCs and LCs–LCk moment ratio diagrams for the homogeneous subdivision All-West, respectively, and Figs. 8 and 9 show the same diagrams for heterogeneous subdivision All-East. The spread around the mean value in Figs. 6 and 7 is much smaller than Figs. 8 and 9. Thus, the assumption that low flows of different stations have the same parent distribution is more acceptable for the All-West subdivision than that for subdivision All-East. Consequently, the quantiles were estimated for individual stations in heterogeneous subdivision All-East but regional quantile estimates were used for homogeneous subdivision All-West.

Fig. 5 LCs–LCk moment ratio diagram for 19 stations in North of Iran

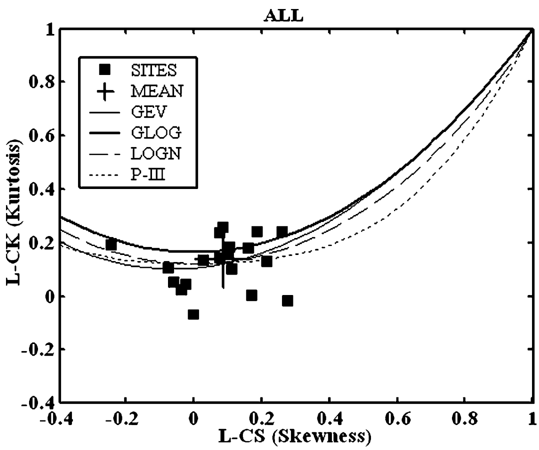


Table 2 Homogeneity measures for study area

Regions	Number of stations	Included stations	H_1	H_2	H_3
ALL	19	1–19	6.77*	0.5***	−1.29***
East	7	1–7	7.54*	1.87**	−0.90***
Middle	5	8–12	4.75*	1.83**	0.18***
West	6	13–19, without St. 17	0.37***	0.46***	−0.41***
All East	7	1–7	7.54*	1.87**	−0.90***
All West	11	8–19, without St. 17	0.71***	−0.37***	−0.92***

*Heterogeneous, **Possibly heterogeneous, ***Homogeneous

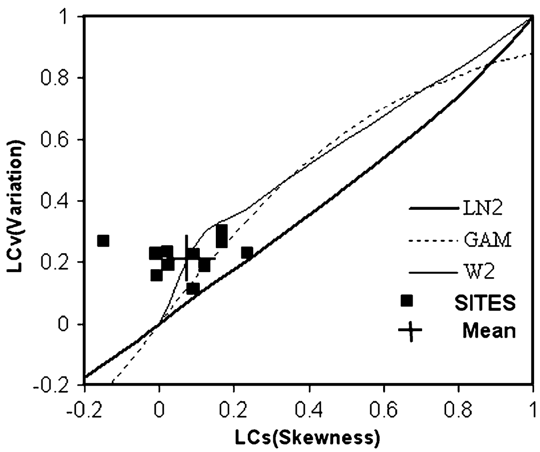
3.4 Quantile Estimation and Goodness-of-fit-test

Quantile estimation is the main task in frequency analysis. There are several frequency distributions used in regional and at-site low flow frequency analysis. Analyzing 20 rivers in Virginia, Tasker (1987) recommended that the 3-parameter Weibull (W3) and Log Pearson distributions to describe the frequency of 7-day annual low flow series. Vogel and Kroll (1989) recommended 2 and 3-parameter LogNormal, Log Pearson and Weibull (W3) distributions for 23 sites in Massachusetts, while, Onoz and Bayazit (1999) fitted Generalized Extreme Value distributions to 16 European rivers.

The Gamma distribution has also been recommended for low flow frequency analysis (Joseph 1970). For Atrak basin in the Northeast of Iran, Nosrati (2003) recommended W3, LP3, LN2, LN3 and Gamma distributions for 7-day low flow frequency analysis. In the study area, we first applied the goodness of fit test for homogeneous subdivisions and then fit a distribution to single sites in the heterogeneous subdivisions.

The goodness-of fit-test measures, Z^{DIST}, (Hosking and Wallis 1993) were calculated using the FORTRAN computer program developed by Hosking (1991) and presented in Table 3. Because the region, as a whole, is not homogeneous, we estimated regional quantiles for the homogeneous subdivision, All West, and estimated quantiles of the sites within heterogeneous subdivision, All East, separately. The quantiles were estimated using the most popular 3-parameter distributions and other 2-parameter distributions fitted to individual satiations.

Fig. 6 LCv–LCs moment ratio diagram for 11 stations in subdivision All-West



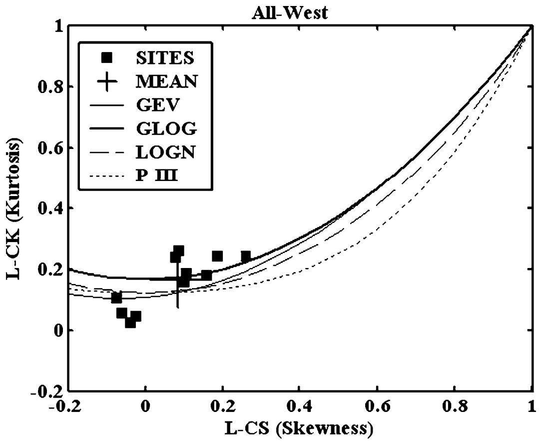


Fig. 7 LCs–LCk moment ratio diagram for 11 stations in subdivision All-West

3.5 At-site Quantile Estimation for Subdivision All-East

From Figs. 8 and 9, it seems that none of the 3-parameter distributions fits to low flow of subdivision All-East. Trying to give a general conclusion on the utility of L-moment ratio diagrams to find a parent distribution, Peel et al. (2001) showed that for heterogeneous samples, the sample average is not useful for selecting parent distribution function. To find the best at-site frequency distribution one can assess the goodness of fit using least-square error. The root mean square error was used as a measure of goodness of fit test for each single station in this subdivision. Table 4 presents quantile estimations of each single site in the subdivision. The reader is referred to Rao and Hamed (2000) for further details on 3-parameter distributions functions.

At-site and regional quantile plots of these stations are also shown in Fig. 10 using plotting position formula $Pi{:}n{=}(i{-}0.35/n)$. The best distribution is that in which the observations lie nearest to the straight line. In this case, the best distributions are mostly 2-parameter distributions, with the exception of Rigcheshmeh station. The atsite and regional quantile estimations of this station are given in Table 5. Based on minimum RMSE, GEV was the best distribution for at site distribution. Figure 10 also indicates a significant

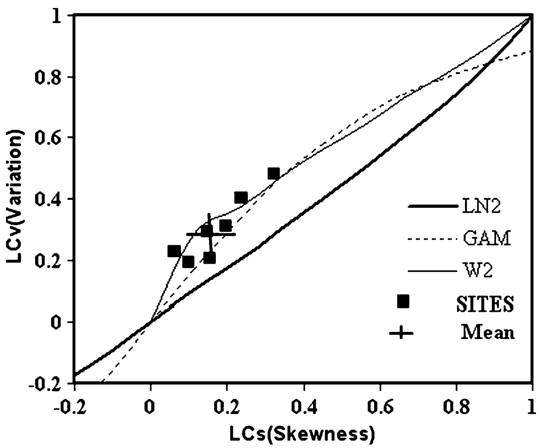


Fig. 8 LCv–LCs moment ratio diagram for 7 stations in subdivision All-East

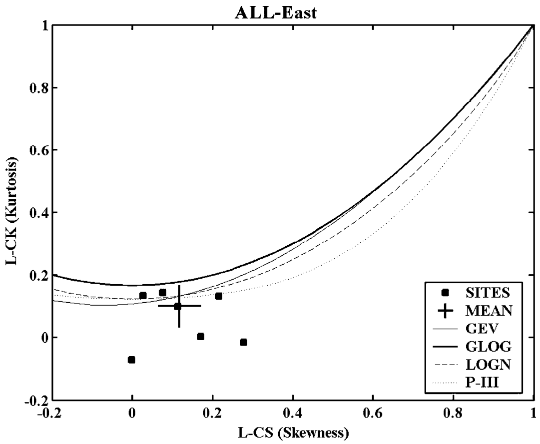


Fig. 9 LCs–LCk moment ratio diagram for 7 stations in subdivision All-East

difference between the at-site and the regional quantile estimation. We therefore can not assume a parent distribution for subdivision All-East.

The result of at site and regional frequency analysis of All-East heterogeneous subdivision also expressed the result of Stedinger and Lu (1995), a key issue where to regionalize 2 parameter distributions, who concluded that the performance of GEV-2 quantile estimators would be better than GEV-3 quantile estimators in semi-arid region, e. g. All-East subdivision, and that 2-parameter at-site Gumbel (EV1) quantile estimators (GEV-2 with $k=0$) often had a smaller RMSE, particularly with smaller sample size, e. g. Alahabad station with sample size=10 years.

3.6 At-site Quantile Estimation for Subdivision All-West

In Section 3 we showed that subdivision All-West is a homogeneous region. In this section we present at-site and regional quantile estimations for the single stations and the region, respectively, through root mean square error measures of goodness of fit. Kalardasht station is given as an example of regional and at site quantile estimation (Table 6). The at-site and regional frequency curves are presented in Fig. 11 using plotting position formula $Pi{:}n=(i-0.35)/n$. While GLOG distribution was identified as a candidate regional distribution, both GAMMA and Normal or 2-parameter Log Normal (LN2) also performed well. 3-parameter Log Normal seemed to be the worst in both regions. Figure 5 shows that the

Table 3 Goodness-of-fit-test measures (Z^{dist}) for study area

Region	GLOG	GEV	LN3	P3	GPAR
ALL	0.68*	−1.88	−1.71	−2.03	−7.04
West	−0.23*	−1.72	−1.59*	−1.75	−4.70
Middle	−0.23*	−0.90*	−0.87*	−1.06*	−3.22
East	1.84	0.13*	0.25*	0.04*	−3.31
All West	−1.22*	−3.10	−2.90	−3.06	−6.08
All East	2.14	0.34*	0.34*	0.01*	−3.41

*The distribution may be accepted as a regional distribution

Table 4 At-site quantile estimations of stations in subdivision All-East (M^3/S)

Sites	Rig Cheshmeh	Varan	Garmrud	Soleymantangeh	Shirgah	Alahband	Gharantalar
Return period (Years)	Best distribution						
	GEV	LN 2	Gamma	Normal	Normal	EV1	LN 2
2	3.55	0.75	0.5	2.74	2.61	0.96	0.55
5	3.10	0.55	0.3	2.5	2.00	0.77	0.42
10	2.51	0.38	0.08	1.64	1.15	0.56	0.2
20	2.21	0.33	0.05	1.5	1.00	0.51	0.19
50	1.83	0.29	0.03	1.28	0.72	0.42	0.14
100	1.39	0.20	0.02	0.77	0.01	0.28	0.01

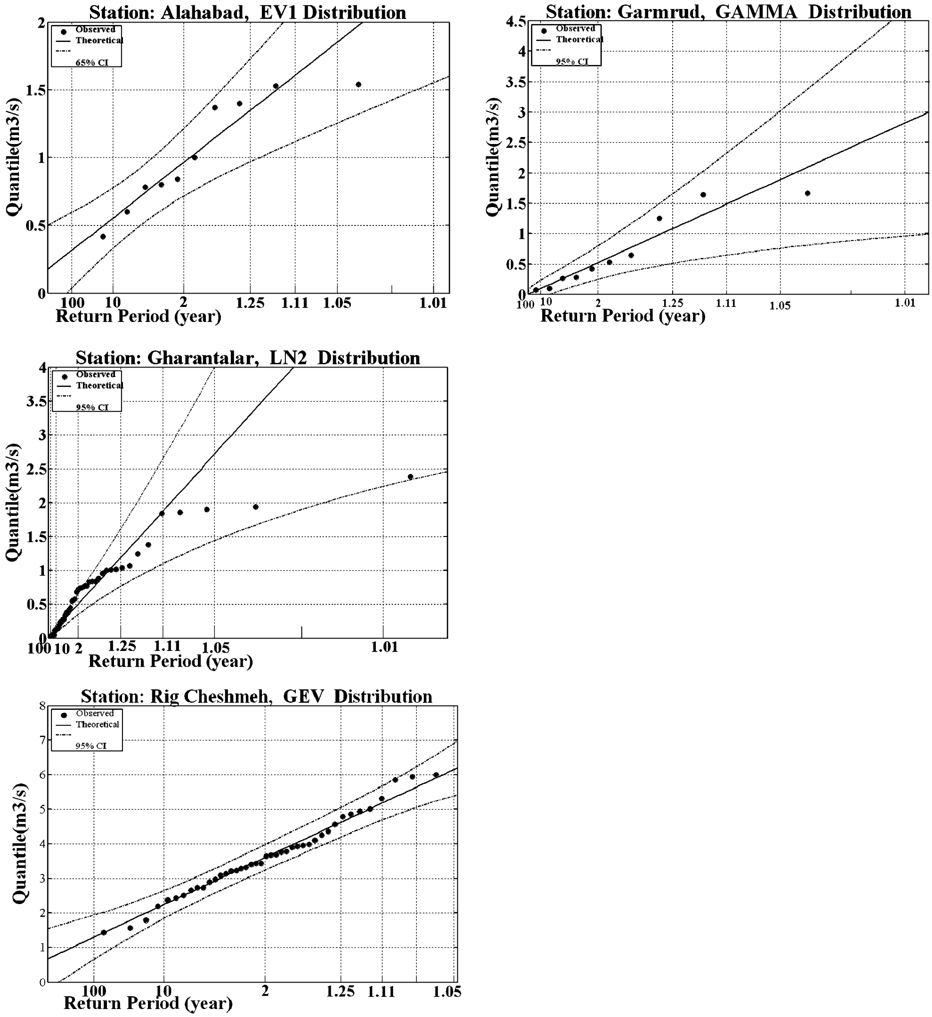


Fig. 10 At site plotting positions and quantile estimations of the stations in subdivision ALL-East

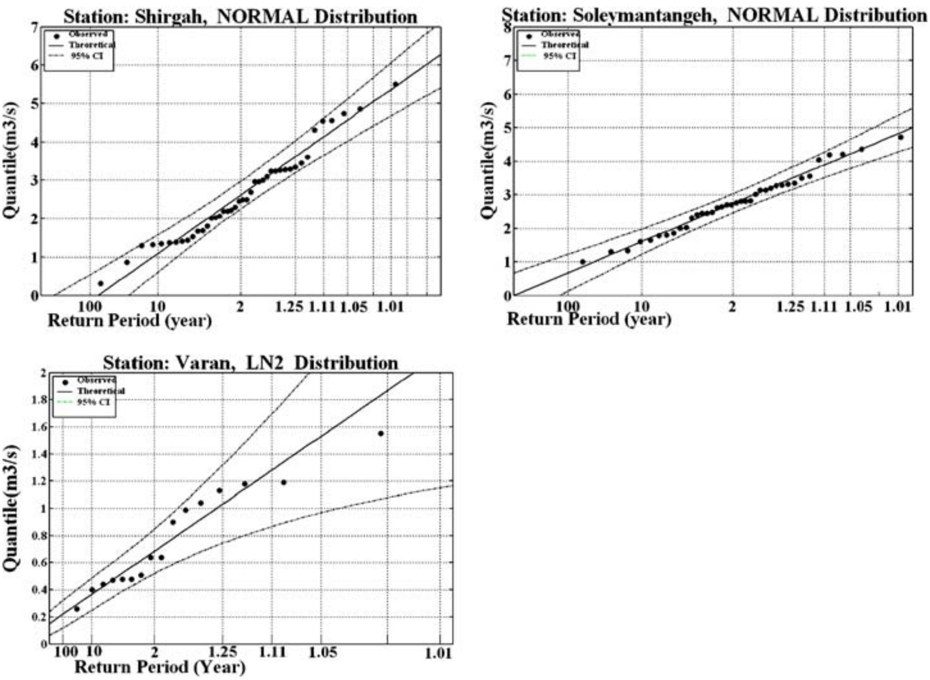


Fig. 10 (continued)

average is located immediately on the GLOG line. The consequent selection of the GLOG distribution as the parent distribution of the All-West subdivision reinforces the conclusion made by Peel et al. (2001) that the sample average is a very good indicator of the parent distribution for homogeneous data.

Although GLOG was selected as the regional parent distribution for the homogeneous ALL-West subdivision, the 2-parameter distributions performed better at individual sites.

It could also be seen that the majority of the low flow events fell within 1.25–10 year-return periods. This suggests that hazardous low flow events with low probability (high return periods of 50–100) have rarely occurred in the region. The difference between at-site and regional quantile estimation with observed value in the upper and lower tails is high. In other words, most of the cumulative distribution function (CDF) models described the observations in the probability range $0.1 \leq p \leq 0.8$ better than outside of this range.

Table 5 Estimated quantiles for Rigchehmeh station in All-East subdivision for different at-site and regional distribution

Return period (year)	Observed	At-site				Regional			
		GEV	III-P	GLOG	LN3	GEV	III-P	GLOG	LN3
2	3.61	3.55^{u}	3.59^{u}	3.60^{un}	3.59^{u}	3.46^{u}	3.46^{u}	3.48^{un}	3.46^{u}
5	3.10	3.10^{n}	3.12^{o}	3.12^{o}	4.63^{o}	3.01^{u}	3.02^{un}	2.97^{u}	2.99^{u}
10	2.10	2.11^{on}	2.13^{o}	2.21^{o}	5.23^{o}	1.65^{u}	1.55^{u}	1.75^{un}	1.52^{u}
20	1.95	2.01^{on}	2.01^{on}	2.04^{o}	5.76^{o}	1.45^{un}	1.35^{u}	1.20^{u}	1.42^{u}

O: overestimate, U: underestimate, N: nearest value

Table 6 Estimated quantiles for Kalardasht station in All-West subdivision for different at-site and regional distribution

Return period (year)	Observed	At-site					Regional				
		GEV	III-P	GLOG	LN3	LN2	GEV	III-P	GLOG	LN3	LN2
2	0.52	0.52^{n}	0.52^{n}	0.52^{n}	0.52^{n}	0.54^{o}	0.55^{o}	0.55^{o}	0.56^{o}	0.55^{o}	0.54^{on}
5	0.46	0.47^{o}	0.45^{u}	0.46^{n}	0.46^{n}	0.46^{n}	0.49^{o}	0.47^{o}	0.47^{o}	0.47^{o}	0.46^{n}
10	0.40	0.37^{u}	0.36^{u}	0.35^{u}	0.36^{u}	0.38^{un}	0.33^{u}	0.32^{u}	0.32^{u}	0.31^{u}	0.38^{un}
20	0.37	0.35^{u}	0.35^{u}	0.34^{u}	0.35^{u}	0.36^{un}	0.30^{u}	0.31^{u}	0.31^{u}	0.30^{u}	0.37^{n}

O: overestimate, U: underestimate, N: nearest value

3.7 Calibration

To assess the goodness-of-fit-test of regional distribution selected for homogeneous subdivision All-West, we implement the average weighted distance (AWD) method of Kroll and Vogel (2002) which measures the differences between sample and theoretical L-moment ratios. The AWD is defined by

$$\mathrm{AWD} = \frac{\sum_{i=1}^{N} n_i d_i}{\sum_{i=1}^{N} n_i} \tag{14}$$

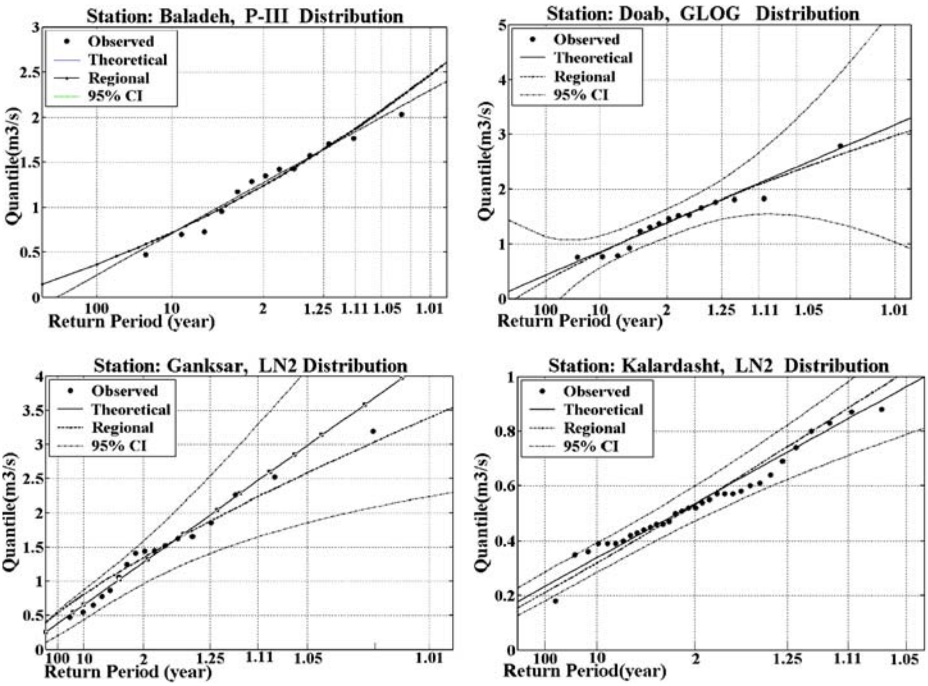


Fig. 11 At site and regional plotting positions and quantile estimations of the stations in subdivision ALL-West

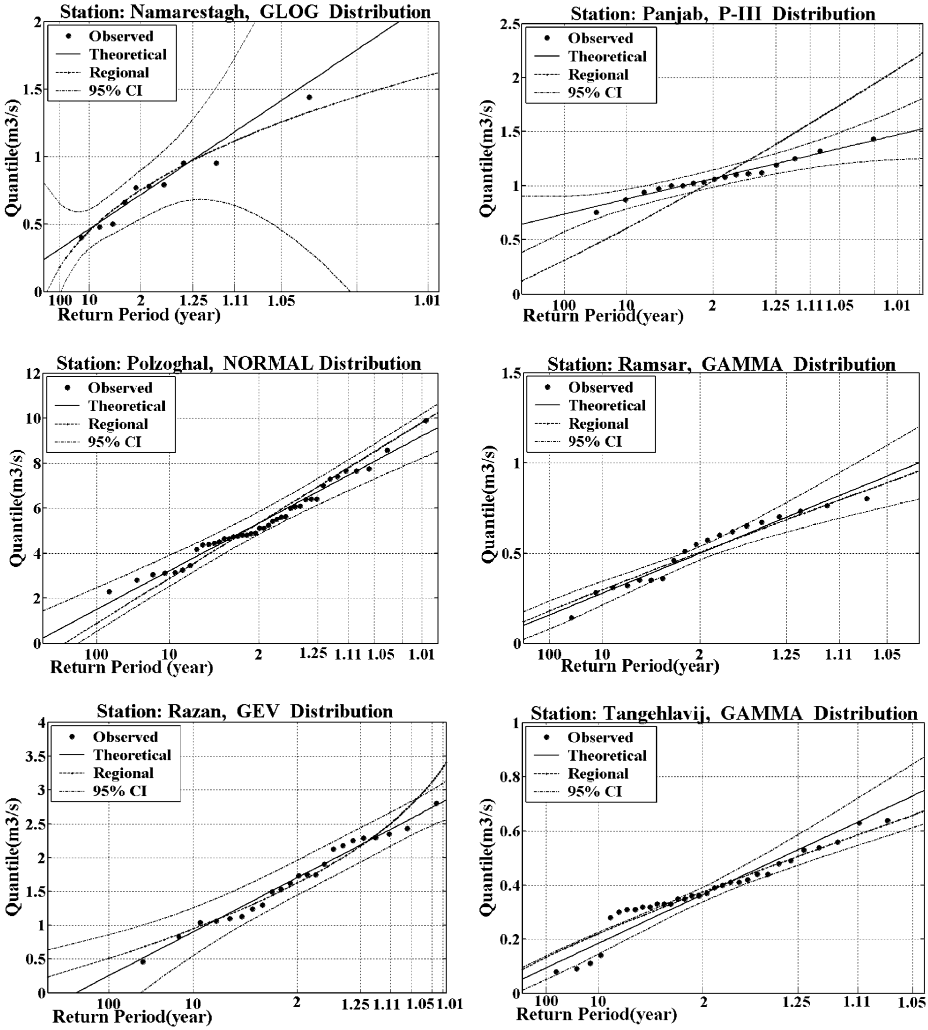


Fig. 11 (continued)

$$d_i = \left|\tau_2\left[\tau_3^0(i)\right] - \tau_2^0(i)\right| \qquad \text{for 2} - \text{parameter distribution}$$

$$d_i = \left|\tau_4\left[\tau_3^0(i)\right] - \tau_4^0(i)\right| \qquad \text{for 3} - \text{parameter distribution}$$

where N is the number of sites in analysis; n_i is the record length at site i; $\tau_k^0(i)$ (k=2, 3 and 4) are the observed or sample L-Cv, L-Cs and L-Ck, respectively; $\tau_2\left[\tau_3^0(i)\right]$ and $\tau_4\left[\tau_3^0(i)\right]$ are theoretical L-Cv and L-Ck values calculated from a distribution corresponding to a given sample L-Cs, respectively. A distribution with the smallest AWD value provides the best-fit to sample data. For the All-West subdivision, AWD is 0.0460, 0.0476, 0.0480 and 0.0496 for GLOG, GN, P3 and GEV distribution, respectively. From AWD, it can be seen that the GLOG is the best regional frequency distribution. It can also be seen that the result of AWD is identical to Z-statistics approach.

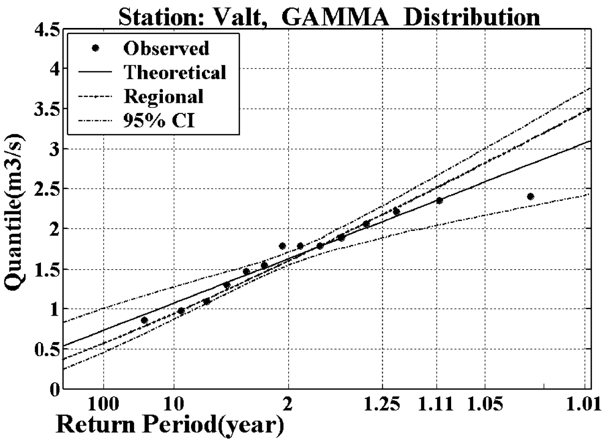


Fig. 11 (continued)

3.8 Investigating Low Flow Relationship with Physical Characteristics of the Watersheds

3.8.1 Development of Regional Annual Low Flow Relationship

For estimation of T-year return period low flow at non-gauged watersheds in subdivision All-West, a set of physiographic and climatic characteristics (called "attributes") of a watershed is required. In this study, four independent attributes, Area, Elevation, Slope and Mean annual Rainfall (MAR) of each watershed were selected. The multivariate regression analysis between low flow quantiles and these attributes indicated that none of the above attributes has significant correlation with quantiles, except for the Area of watershed. Thus, the following relationships were developed for All-West subdivision. All the equations are significant at 1% significant level. No significant relationship was found for 100-year return period.

$$\overline{Q} = 0.033A^{0.58} \tag{15}$$

$$Q_2 = 0.03A^{0.6} \tag{16}$$

$$Q_5 = 0.029A^{0.57} \tag{17}$$

$$Q_{10} = 0.029A^{0.52} \tag{18}$$

$$Q_{25} = 0.027A^{0.51} \tag{19}$$

$$Q_{50} = 0.01A^{0.49} \tag{20}$$

3.8.2 Relationship Between Frequency Distributions and Watershed Characteristics

The results obtained that variation of geography greatly influence the frequency distribution. The histograms in Fig. 12 show the relationship between 2 and 3 parameter distribution functions and some characteristics of the watersheds in the study area. There is not a significant height-rainfall gradient in the region. Although the number of 2-parameter distributions are more frequent in the region, the proportion of them is higher in the low-elevated region (175<height<350 m, Fig. 12a). The higher percentage of 3-parameter distribution in middle and upland rivers suggests that snowmelt may be a major mechanism of low flow generation. A slightly different conclusion can be inferred from the relationship between rainfall and frequency distributions (Fig. 12b). The figure shows that 2-parameter distributions are more appropriate for the stations with higher rainfall. The lack of significant rainfall-height gradients prevents uniform conclusions on the combined effect of height and rainfall on low flow characteristics in this region similar to those made by Vogel and Kroll (1990) who demonstrated that annual precipitation and basin relief are significant parameters in the calculation of low flows. Comparing the relationship between rainfall and the type of 3-parameter distributions, Zaidman et al. (2003) concluded that, for short duration low flows such as 7-day low flows, the PIII and GLOG distributions are favored by watersheds with low rainfall while the GEV distribution is better for watersheds with high rainfall. However, they did not check 2-parameter distributions. In contrast, most of the watersheds with low rainfall explored as part of this study had 3-parameter distributions. This may be the reason of significant correlation between low flow quantile and watershed area rather than other catchment properties. Yue and Wang (2004) also concluded that the watershed area is the only factor which is related to PWMs.

It can also be seen that no significant conclusion can be drawn on the relationship between area and the type of the distribution (Fig. 12c). However, this may be due the small number of watersheds in this study. We need to use more watersheds to obtain a reliable conclusion. The comparison between sample size and the type of the distribution (Fig. 12d) also needs more investigation.

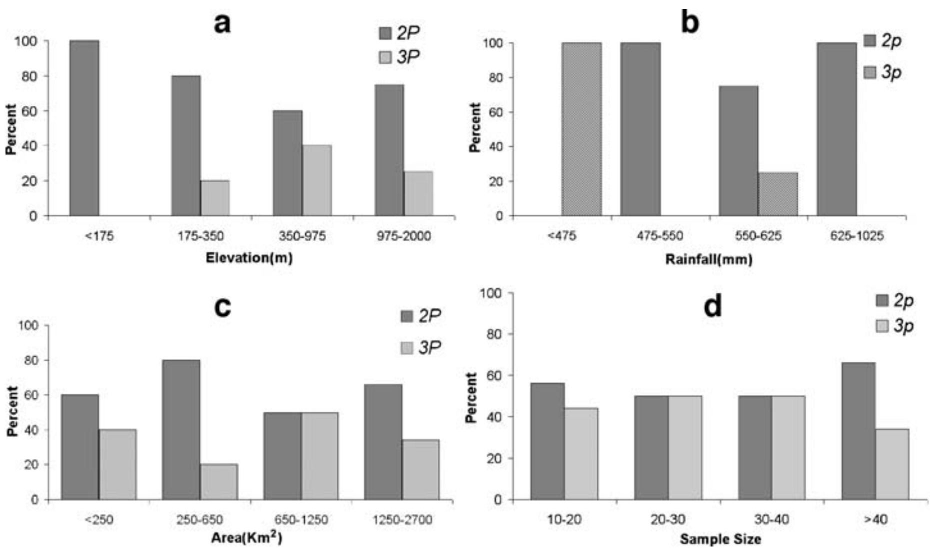


Fig. 12 Comparison of the percent of fitted 2 and 3-parameter distributions (2p: 2-parameter distribution, 3p: 3-parameter distribution) related to watershed characteristics, **a** Elevation (m), **b** Rainfall (mm), **c** Area (km^2), **d** Sample Size

4 Conclusions

Nineteen unregulated gauged sites of Mazandaran province in the north of Iran with a continuous low flow record of at least 10 years were analyzed using L-moments to study regional homogeneity and frequency analysis. Based on homogeneity measure, H_1, the whole region is heterogeneous. This heterogeneity is due to different climatic conditions from west regions eastward. Therefore, the region was divided into three subdivisions in order to find homogeneous regions. Two of the three subdivisions with seven and five stations, were identified as homogeneous while the third region, located in the semi-arid eastern region was identified as heterogeneous. This may be a result of different climatic and geographic differences between the west and east regions in the north of Iran. The two homogeneous subdivisions were then merged to each other to form a larger homogeneous region. The comparative regional and at-site frequency analyses showed that the GLOG distribution is the best distribution according to goodness of fit test measure, Z^{DIST}, and AWD for the homogeneous All-West subdivision. Although no single distribution is suggested as the parent distribution for the heterogenous All-East subdivision, the 2-parameter LN2 distribution performs better than other distributions for at-site frequency analysis.

In conclusion, the area of the watershed can be considered as the main factor of low flow variation in the north of Iran. It is in agreement with the results of other studies such as Vogel and Kroll (1989) or Kroll and Vogel (2002) for low flow frequency analysis and with Rao and Hamed (1997), Potter and Lettenmaier (1990), and Vogel et al. (1993) for floods. Based on Yue and Wang (2004), it should also be noted that low flow obey a simple scaling law and the Index method seems to still be a reliable method for regional frequency analysis. However, the simple Scaling laws of low flows in Iran need to be carefully investigated in future studies.

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References

ARIDE (1999) Methods for regional classification of streamflow drought series: The EOF method and L-moments. Technical report, no, 2

Chen YD, Huang G, Shao Q, Xu CY (2006) Regional analysis of low flow using L-moments for Dongjiang basin, South China

Dingman SL, Lawlor SC (1995) Estimating low flow quantils from drainage basin characteristics in New Hampshire and Vermont. Water Resour Bull 31(2):243–256

Durrans SR, Tomic S (1996) Regionalization of low flow frequency estimation: an Alabama case study. Water Resour Bull 32:223–237

Greenwood JA, Landwahr JM, Matalas NC, Wallis JR (1979) Probanility weighted moments: definition and relation to parameters of several distributions expressible in inverse form. Water Resour Res 15(5):1049–1054

Hosking JRM (1986) The theory of probability weighted moments. Res. Rep. RC 12210, IBM Research Division, Yorktown Heights, NY. 10598

Hosking JRM (1990) L-moments: analyzing and estimation of distributions using linear combinations of order statistics. J R Stat Soc B 52:105–124

Hosking JRM (1991) Fortran routines for use with the method of L-moments, Version 2, Res. Rep. RC 17097, IBM Research Division, York Town Heights, NY 10598

Hosking JRM, Wallis JR (1987) Parameter and quantile estimation for the Generalized Pareto distribution. Technometrics 29(3):339–349

Hosking JRM, Wallis JR (1993) Some statistical useful in regional frequency analysis. Water Resour Res 29 (2):271–281

Hosking JRM, Wallis JR, Wood EF (1985) Estimation of the generalized extreme value distribution by the method of probability weighted moments. Technometrics 27:251–261

Joseph ES (1970) Probability distribution of annual drought. J Irrig Drain 94(4):461–474

Kroll CK, Vogel RM (2002) Probability distribution of low streamflow series in the United States. J Hydrol Eng 7(2):137–146

Kumar R, Chatterjee C, Kumar S, Lohani AK, Singh RD (2003) Development of regional flood frequency relationships using L-moments for Middle Ganga Plains Subzone 1(f) of India. Water Resour Manag 17:243–257

Lim YH, Lye LM (2003) Regional flood estimation for non-gauged basins in Sarawak, Malaysia. Hydrol Sci J 48:79–94

Nosrati K (2003) Regional frequency analysis of low-flow in arid and semi arid regions, Case study: Atrak river basin. MSc Thesis, University of Tehran (in Persian)

Onoz B, Bayazit M (1999) GEV-PWM model for distribution of minimum flows. J Hydrol Eng 4(3):289–292

Parida BP, Kachroo RK, Shrestha DB (1998). Regional flood frequency analysis of Mahi-Sabarmati basin (Subzon 3-a) using index flood procedure with L-moments. Water Resour Manag 12:1–12

Pearson CP (1995) Regional frequency analysis of low flows in New Zealand rivers. J Hydrol NZ 33:94–122

Peel MC, Wang QJ, Vogel RM, McMahon TA (2001) The utility of Lmoment ratio diagrams for selecting a regional probability distribution. Hydrol Sci J 46(1):147–155

Potter KW, Lettenmaier DP (1990) A comparison of regional flood frequency estimation methods using a resampling method. Water Resour Res 26(3):415–424

Rao AR, Hamed KH (1997) Regional frequency analysis of Wabash river flood data by L-moments. J Hydrol Eng 2(4):169–179

Rao AR, Hamed KH (2000) Flood frequency analysis. CRC, Boca Raton, FL

Rodrigo FS, Esteban-Parra MJ, Pozo-Vázquez D, Castro-Diez Y (1999) A 500-year precipitation record in southern Spain. Int J Clim 19:1233–1253

Stedinger JR, Lu LH (1995) Appraisal of regional and index flood quantile estimators. Stoch Hydrol Hydraul 9(1):49–75

Stedinger JR, Vogel RM, Foufoula-Georgiou E (1993) Frequency analysis of extreme events. In: Maidment DR (ed) Hand book of hydrology. McGraw Hill, New York, NY, pp 18.1–18.66

Tasker GD (1987) A comparison of methods for estimating low flow characteristics of streams. Water Resour Bull 23(6):1077–1083

Tasker GD (1972) Estimating low-flow characteristics of streams in Southeastern Massachusetts from maps of ground-water availability, U. S. Geol. Surv. Prof. Pap. 800-D, D217–D220

Vogel RM, Fennessey NM (1993) L-moment diagram should replace product moment diagram. Water Resour Res 29(6):1745–1752

Vogel RM, Kroll CN (1989) Low-flow frequency analysis using probability-plot correlation coefficients. J Water Resour Plan Manage 115(3):338–357

Vogel RM, Kroll CN (1990) Generalized low flow frequency relationships for non-gauged sites in Massachusetts. Water Resour Bull 26(2):241–253

Vogel RM, Wilson I (1996) The probability distribution of annual maximum, minimum and average streamflow in the United States. J Hydrol Eng 1(2):69–76

Vogel RM, McMahon TA, Chiew FHH (1993) Flood flow frequency model selection in Ausralia. J Hydrol 146:421–449

Yue S, Wang CY (2004) Scaling of Canadian low flows. Stoch Environ Res Risk Assess 18:291–305

Zaidman MD, Keller V, Young AR, Cadman D (2003) Flow-duration-frequency behavior of British rivers based on annual minima data. J Hydrol 277:195–213

Zareiy AR (1999) An evaluation of the methods of regional frequency analysis of low flows in Mazandaran basin, MSc Thesis, Isfahan University of Technology (in Persian)